Relativity

01/10/98

Special Relativity
1:Introduction,
2:Special Relativity,
3:Relativistic kinematics,
4:Relativistic dynamics,
5:Energy and space-time,
6:Experitmental verifactions,
7:Paradoxies, general relativity and more!

Reading,
W.G.V Rosser, "Introductory Special Relativity", BJL: QC 173.65 R8
W Rindler, "Introduction to Special relativity", BJL: QC 173.65 R5
M. Alonso, E.J. Fenn "Physics" (chpt, 19), BJL: qc 21.2 A4

Special Relativity

1 Introduction

1.1 Mechanics

Mechanics deals with space and time.

Space:-
We assume a unique system of coordinates exist which is independent of any object in the universe (absolute space, postulated by Newton). We can have a coordinate system of any object in a reference frame, **S**, and refer to these in another frame, **S'**, by,

fig 1

In fig. 1, x'=x-alpha, y'=y-beta, and z'=z-gamma

Time:-

We assume a time scale independent of any event (absolute time, Newton).

We can relate time by **t'=t-T**, where **S'** has it's time origin, **T**, different to that in **S**.

Classical Mechanics:-

Assumes a signal of infinite velocity to synchronise clocks!

1.2, The Galilean Transformations

fig 2

Galilean Transformation:

(From fig. 2)

Moving to Stationary	Stationary to Moving
$x'=x-v.t$	$x=x'+v.t$
$y'=y$	$y=y'$
$z'=z$	$z=z'$
$t'=t$	$t=t'$

Note, at **t=0** the origins coincide.

Velocities	Accelerations
$u'_x=u_x-v$	$a'_x=a_x$
$u'_y=u_y$	$a'_y=a_y$
$u'_z=u_z$	$a'_z=a_z$

This gives rise to the concept of inertial reference frames. In inertial frames accelerations are seen to be the same in **S** and **S'**, even if **S'** is moving with respect to **S**.

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1.3 Electromagnetism

Take a charge moving in an electric and magnetic field.

(fig 1)

The force on the moving charge in frame **S** is,

$$\underline{\mathbf{F}}=\mathbf{q}.(\underline{\mathbf{E}}+\underline{\mathbf{v}}\ )$$

If the observer moves with a velocity $\underline{\mathbf{v}}$ in **S'** the charge is at rest in **S'**.

(Since $\underline{\mathbf{v}}'=0$, $\underline{\mathbf{v}}'$ is the velocity of the particle in **S'**).

$$\underline{\mathbf{E}}'=\underline{\mathbf{E}}+\underline{\mathbf{v}}$$

If $\underline{\mathbf{v}}$ is parallel to (x) and (x'),

$$\mathbf{E'_x=E_x}$$
$$\mathbf{E'_y=E_y+\underline{v}.\underline{B}_z}$$
$$\mathbf{E'_z=E_z+\underline{v}.\underline{B}_y}$$

$$\mathbf{B'_x=B_x}$$
$$\mathbf{B'_y=B_y+(v.E_z/c\acute{y})}$$
$$\mathbf{B'_z=B_z-(v.E_y/c\acute{y})}$$

Unlike Galilean transformations, these equations are not <u>reciprocal</u>. Since taking, e.g., E'_z.

$$\mathbf{E_z=(E'_z-\underline{v}.B'_y)/(1-(v\acute{y}/c\acute{y}))}$$

Maxawell's equations are not invariant (small differences in going from one frame to another). Maxwell's equations predict the velocity of light to be **cñv**.

<u>1.4 Optics</u>

It was postulated that light needed a medium for propagation this was called the *Ether*. There were several experiments to test this;

a)<u>Fizeau's experiment (1851)</u>

There was a tube of water flowing a velocity **v**. Light was shone through in the same direction as the flow of water.
If the Galilean transformation was correct the velocity of the light relative to the tube would be,

$$\mathbf{c'=c/n+v}$$

where **n** is the refractive index of the water. It was found that,

$$\mathbf{c'=c/n+v.(1-1/n\acute{y})}$$

The factor of **v** was called the *Ether drag coefficient.*

b)<u>The Michelson-Morely experiment (1881)</u>

(fig 2 for schematic of apparatus)
Using the Galilean transformations,

$$\mathbf{L'=PM_1=PM_2}$$

Mirror 1:

$$t_1=L'/(c-v)+L'/(c+v)$$
$$=2L'c/(cý-vý)$$
$$=2(L'/c)/(1-(vý/cý))$$

Mirror 2:

$$t_2=2L'/(cý-vý)^{\wedge}.5$$
$$=2(L'/c)/(1-vý/cý)^{\wedge}.5$$

Experiment:
NO interferenct pattern was found. This suggests that $t_1=t_2$.

2 Special Relativity

2.1 Postualtes

Between 1899 and 1904 PoincarŠ expressed views that only relative motion matters.

Einstein's Postulates (1905)

1) According to the principal of relativity the laws of physics including those of mechanics, optics, electromagnetism and nuclear physics are the same in all inertial reference frames. Numerical values of quantities can be different, no acceleration **(-> general relativity)**
2) According to the principal of the constantcy of the velocity of light the speed of light in empty space has the same numerical value in all intertial reference frames.

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2.2 The Lorentz transformations

fig 1

Two reference frames. **S** and **S'**. Both are parallel in x direction. At t=0 a flash of light is emitted at the common origin of the two observers (**O** and **O'**). The flash is at a point (or ***event***) A. A has coordinates (x,y,z,t) in **S**, and (x',y',z',t') in **S'**.

After a time t, observer O will note that the light has reached point A and will write,

$$r=C.t$$

Where C is the velocity of light.

Since,

xý+yý+zý=rý

Therefore,

xý+yý+zý=Cý.tý (1)

Since the observer O’ will note,

r’=C.t’

or

x’ý+y’ý+z’ý=Cý.t’ý (2)

We need to obtain a transformation relating equations (1) and (2).
The symmetry of the problem suggests;

y’=y
z’=z

Also since **OO’=v.t** (OO’ is the distance between observers) for observer **O**, it must be that **x=v.t** for **x’=0** (see fig 1 again). Since **x’=0** is the point **O’**.

This suggests,

x’=*y*.(x-v.t)

Where *y* is a constant which has to be determined.

We may also assume,

t’=a.(t-b.x)

Where **a** and **b** are constants which have to be determined.

Note: For Galilean transformations
y=a=1 & b=0

Now, substituting into (2) gives,

***y*ý.(xý-2.v.x.t.+vý.tý)+yý+zý=Cý.aý.(tý-2.b.t+bý.xý)**

or

(*y*ý-bý.aý.Cý).xý-2.*y*ý.v-bý.aý.Cý).x.t+yý+zý=(aý*y*ý.(vý/Cý).Cý.tý)

The result must identical to (1). Therefore,

***y*ý-bý.aý.Cý=1**
***y*ý.v-b.aý.Cý=0**
aý-*y*ý.(vý/Cý)=1

Which gives,

y=a=1/(1-(vý/Cý))^«
b=v/Cý

We can now obtain the Lorentz transformations,

x’=(x-v.t)/((1-(vý/Cý))^«)
=y.(x-v.t)
y’=y
z’=z
t’=(t-(v.x/Cý))/(1-(vý/Cý))^«)
=y.(t-(v.x/Cý))

These are the full set of the Lorentz transformatons.

In general however,

r’=y.(r-v.t)
t’=y.(t-(v.r/Cý))

For the Inverse Lorentz transformation, we need to exchange x by x’, etc. and t by t’ and we have to replace **v** by **-v**.

For v<<C => Lorentz transformations are equivalent to Galilean transformations. But, for something like cosmic rays or in high energy accelerators, or in electrons orbiting atoms, we need to use the Lorentz transformations.

fig 2

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3 Relativistic Kinematics

3.1 Lenght Contraction

fig 1
(Two cartesian frames, S & S’, a fish is travelling at v in S and is at rest in S’)

The length of an object has to be determined by recording the positions of the two end points simultaneously (except when the object is at rest).

In S, $L=x_b-x_a$
In S’, $L'=x'_b-x'_a$
from Lorentz transformations,

x’$_a$=(x$_a$-v.t)/(1-(vý/cý))^«

x'_b=(x_b-v.t)/(1-(vý/cý))^«

Therefore,

L'=x'_b-x'_a
=(x_b-x_a)/(1-(vý/cý))^«

L=L'.(1-(vý/cý))^«

Since, **(1/(vý/cý))^« < 1** we have **L < L'**.

Therefore the length of an object is measured to be shorter when the object is in motion relative to the observer than when the object is a rest relative to the observer.

i.e. L_{motion}< L_{rest}

<u>3.2 Time Dilation</u>

Imagine that in the frame **S** we have a clock beating in one second intervals. The clock is stationary in **S** (i.e. the time interval in **S** is $T=t_1-t_2$, for $x_2=x_1=x$).

For an observer in **S'** (in motion with respect to **S**) the time interval of the clock in **S** is given by the Lorentz Transformation.

t'_1=(t_1-(v.x/cý))/(1-(vý/cý))^«

t'_2=(t_2-(v.x/cý))/(1-(vý/cý))^«

(note, same x in this case, as before it was the same t)

T'=t'_2-t'_1=(t_2-t_1)/(1-(vý/cý))^«
T'=T/(1-(vý/cý))^«

Again **(1-(vý/cý))^« < 1** we have **T' > T**.

Therefore, processes appear to take a longer time when they occur in an object in motion relative to the observer than when the object is at rest relative to the observer.

i.e. T_{motion}> T_{rest}

<u>3.3 Simultaneity and Order of events</u>

fig2
(Cartesian Frames S & S'. Two points in S, A and B. A is at and t_1,

and B is at x_2 and t_2. A reference frame S' moving at v)

Suppose the two events A and B occur simultaneously in **S**. If in **S** events would occur at a time interval **dt.**

$$dt'=(dt-v.dx/c^2)/(1-(v^2/c^2))^{1/2}$$

However simultaneity in **S** means that **dt=0**. Therefore,

$$dt'=(-v.dx/c^2)/(1-(v^2/c^2))^{1/2}$$

i.e. A and B are <u>not</u> seen as simultaneous events by an observer in **S'**. Singals take different times to reach the observer in **S'**.

$$dt'=t'2-t'1 < 0$$
=> observer O' sees B before A

If dx=0 then (and only then) dt'=0.

<u>Question</u>: is it possible possible that, if events A and B occur in that sequence in S, they can be observed as B and A in S'.

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<u>Answer</u>: we know that in S',

$$t'_2-t'_1=(t_2-t_1)-(v/c^2).(x_2-x_1)/(1-(v^2/c^2))^{1/2}$$

A is observed before B in frame S' as long as t2'-t1' is positive.

$$(t_2-t_1) > (v/c^2).(x_2-x_1)$$

$$(x2-x1)/(t2-t1) < (c^2/v)$$
(See section 5.2)

<u>Note</u>: If v is negative and that dt=0 then t'2 > t'1 (t2 refers to event B and t1 refers to A).

<u>3.4 Synchronization of spatially separated clocks</u>

<u>Problem</u>: how can an astronaut synchronise his clock on the spaceship to keep the same time as on the earth ?

<u>Einstein's prescription</u>:
- send a radio signal from earth to space ship.
- astronaut sends signal back without time delay and notes time on spaceship clock when signal arrived.
- bas station on earth measures times of sending out (t1) an recieving (t2) the signal, estimates that signal reached the

astronaut at a time «(t1+t2) and transmit this information to the astronaut
- astronaut can set his clock to agree with clock on earth.

Note: There is no infinitley dast transformission of information (fastest possible speed is speed of light). Looking at the stars is looking into the past!

Questions,
1) In a particular intertial reference frame a rod of length one meter is at rest and inclined at an angle of 45 degrees to the axis. A second intertial frame moves with a relative speed at 0.8 c in the x direction. What is the length of the rod and it's angle of inclination to the x axis in this frame ?

2)

Answer

1) angle=59 degrees.

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3.5 The velocity transformations

Particle in system S and it is travelling with a velocity **u**, having components (u_x,u_y,u_z). How will this look from the frame S' (which is moving at a velocityt v relative to S).

Fig One

In frame S,

$$\mathbf{u_x=dx/dt}$$
$$\mathbf{u_y=dy/dt}$$
$$\mathbf{u_z=dz/dt}$$

In framse S',

$$\mathbf{u'_x=dx'/dt'}$$
$$\mathbf{u'y=dy'/dt'}$$
$$\mathbf{u'_z=dz'/dt'}$$

We can find these by differentiating the lorentz transformations.

Hence with **dx=u.dt**,

dx’=(dx-v.dt)/(1-vý/cý)^«
=((u_x-v)/(1-vý/cý)^«).dt

dy’=dy
dz’=dz

dt’=(dt-(v.dx/cý))/(1-vý/cý)^«
=((1-(v.u_x/cý))/(1-vý/cý)^«).dt

Dividing dx’ by dt’ gives,

u'_x=(u_x-v)/(1-(v.u_x/cý))
u'_y=(u_y.(1-vý/cý)^«)/(1-(v.u_y/cý))
u'_z=(u_z.(1-vý/cý)^«)/(1-(v.u_z/cý))

For u_x=u and u_y=u_z=0,

u’=(u-v)/(1-(v.u/cý))

Note:
dy’=dy
u'_y<>u_y
This is because dt’<>dt.
If v<<c : u'_x=u_x-v which is of the form of classical mechanics.

The inverse relations are obtained by interchanging **u’** and **u** and replacing **v** by **-v**. These transformations explains the Michelson and Morely experiment.

Explanaton of Michelson and Morely experiment.

Suppose of particle is a photon travelling with u_x=c (or with the speed of light) what be the speed of light as seen by the observer in S’ ?

u_x=c

u_x’=(u_x-v)/(1-(v.u_x/cý))
=(c-v)/(1-(vc/cý))
=((c-v)/(c-v)c

u_x’=c

So, the speed of light is the same for all observers.

3.6 The acceleration transformations

In the frame S the acceleration **a** of a particle is also a vector with components,

$$a_x=du_x/dt$$
$$a_y=du_y/dt$$
$$a_z=du_z/dt$$

Similarly **a’** in S’ is,

$$a'_x=du'_x/dt'$$
$$a'_y=du'_y/dt'$$
$$a'_z=du'_z/dt'$$

Differentiating **u’** gives,

du’=((1-vý/cý)/(1-v.u/cý)ý).du

Hence,

a’=du’/dt’=(((1-vý/cý)^(3/2)/(1-v.u/cý)ü).du/dt

Where **a=du/dt**.

Note: for v<<c then a’=a

The Transformations for (1-u’ý/cý)^« and (1-uý/cý)^«

(1-u’ý/cý)^«=((1-vý/cý).(1-uý/cý))^«/(1-(c.u/cý))

(1-uý/cý)^«=((1-vý/cý).(1-uý/cý))^«/(1+())

u’ is the total velocity of a particle in S’ (u’ý=ux’ý+uy’ý+uz’ý).

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3.7 Interpretation of optical experiments

a) Fizeau’s experiment

Description: Light shone down a capillary tube which has water flowing along it.

Consider a frame of reference S’ which is moving at the same velocity as the water flow in the observers frame, S. So that the water is at rest in S’. The velocity of light in the water is U'_x, where U'_x=c/n (n is refractive index). We now write the trnsformation,

U_x=(U'_x+v)/(1+(v.U_x/cý))

=(c/n).(1+(n.v/c)).((1+(v/n.c))^-1)

Expanding this by the binomial theorem (excluding terms **vý/cý**).

U_x=(c/n).(1+(n.v/c)).(1-(v/n.c))
=(c/n)+v.(1-(1/ný))
(all approximate)

b) <u>The Doppler Effect</u>

Consider two reference frames, S and S', with obersevers O and O' respectivley. S' is moving away from S in the positive x direction at a velocity v. A point P exists in S'. Electromagnetic waves are emitted by P. To observer O these waves have the form,

sin(k(x-ct))

to observer O' they are,

sin(k'(x'-ct'))

The principal of relativity requires,

k(x-ct)=k'(x'-ct')

From the Lorentz transformations we have,

k(((x'+vt')/((1-vý/cý)^«))-c((t'+(vx'/cý))/((1-vý/cý)^«)))=k'(x'-ct')

k'=k (1+v/c)/((1-vý/cý)^«)
=k((1-v/c)/(1+v/c))^«

Remembering that *w*=c.k and *v*=*w*/2pi. Hence we write,

***v*'=*v*.((1-v/c)/(1+v/c))^«**

For v<<c (using binomial theorem),

***v*'=*v*.((1-«(v/c))/(1+«(v/c)))**
=*v*.(1-v/c)

This agrees with the classical expression for the doppler effect.

Example
Spectra of distant galaxies are red shifted. The red shift increases with distance suggesting a greater recessional velocities. This gives the idea of an expanding universe and leads to **Hubble's Law**. **v=HR**
(H=2.32 10^-18 1/s).

Simultaneity - see note book!

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4 Relativistic Dynamics

4.1 Transformations of mass

The classical definition of momentum is,

$$\mathbf{\underline{p}=m.\underline{u}}$$

From the principal of relativitywe know that the law of conservation of linear momentum must also hold in relativistic mechanics. However, in combining the definiton of classical momentum (see above) with the Lorentz transformations we find that the law of conservation of linear momentum is violated. This means that we need a new definition of momentum (relativistic momentum). Linear momentum can be conserved if the mass is redefined such that when a particle is moving with velocity **u** it's mass is,

$$\mathbf{m=m_o.\gamma}$$

Where m_o is the mass of the particle measured in the interial frame in which it is at rest (the *rest mass*). Experiments have confirmed this new definition of the mass, m. (e.g. measurements in particle experiments).
N.B. It is more difficult to accelerate a particle as it moves faster relative to the observer.

4.2 Relativistic Momentum and the definition of force.

In relativistic mechanics the momentum of a particle moving with velocity **u** is the following,

$$\mathbf{\underline{p}=m.\underline{u}=m_o.\underline{u}.\gamma}$$

It has components,

$$p_x=m_o.u_x/(1\text{-}(u_x\text{ý}/c\text{ý}))\text{^«}$$
etc

The force acting on a particle is given by the following definition,

$$\underline{F}=d\underline{p}/dt$$

Therefore, in relativistic mechanics,

$$\underline{F}=d/dt\ (m\underline{u})=m.du/dt + u.dm/dt$$

Note,
the first term is the result from classical (or Newtonian) mechanics.

a) Rectilinear motion

$$F=d/dt\ (m_o.u.y)=(m_o.y\text{ü}).du/dt$$

b) Uniform circular motion ($|\underline{u}|$=const)

$$\underline{F}=(m_o.y).d\underline{u}/dt$$
$$F=(m_o.y).(u\text{ý}/R)$$

This is a centripetal force (radius R).

c) General case of curvilinear motion
Consider a particle P, moving along a curved path. It has a tengential acceleration a_T, there is also an acceleration normal to the path, a_N. The resultant acceleration is **a**.

$$\underline{F}_T=(m_o.y\text{ü}).\underline{a}_T$$
$$\underline{a}_T=du/dt\ \text{(tangential acc.)}$$

$$\underline{F}_N=(m_o.y).\underline{a}_N$$
$$\underline{a}_N=u\text{ý}/R\ \text{(centripetal acc.)}$$

Note
The force is not parallel in the relativistic formalism to the acceleration.

5. Energy and Space-Time

5.1 Work and Kinetic Energy

As in Newtonian mechanics the work done on a particle will be defined as the scaler product of force and displacement.

$$dW=\underline{F}.d\underline{l}$$

If it is assumed that all this work goes into increasing the kinetic energy of the particle, then (where dT is the change in kinetic energy),

dT=F.dl

The rate of increase of kinetic energy T is the following,

dT/dt=F.dl/dt=F.u

By definition,

F=d/dt (mu)

where,

m=m_o.*y*

Hence,

dT/dt=(d/dt (mu)).u
=m.(du/dt)u + (dm/dt)u.u

dT/dt=(m_o.*y*).u.(du/dt)+m_o.(du/dt).((uü/cý).*y*ü)
=m_o.u.(du/dt).(1-(uý/cý)+(uý/cý)).*y*ü
=(m_o.u.*y*ü).(du/dt)
=d/dt (m_o.cý.*y*)

Integrating gives now,

T=($m_o$cý.*y*)+C_1

T=0 when u=0 => -m_o.cý

T=(m_o.cý.*y*)-m_o.cý

T=m.cý-$m_o$cý

Thus,

E=T+m_o.cý
E=mcý
=m_o.cý.*y*

Where is the total energy of a free particle and m.cý is generally called the rest mass energy of a free particle.

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Note:

If we expand the following kinetic energy term,

T=m$_o$cý.[(1/(1-uý/cý)^«)-1]

by the binomial theorem,

T=m$_o$cý.[1+«(uý/cý)+(3/8).(u^4/c^4)+...]-m$_o$cý
=«u$_o$.uý+(3/8).m$_o$.(u^4/c^4)+...

Now, if u<<c then,

T=«m$_o$uý
(approximately)

which is in agreements with Newtonian mechanics.

<u>Also</u>,
If we write the energy in the form,

T=m.cý-m$_o$.cý
T=(m-m$_o$).cý
T=dm.cý

This shows that associated with a change in the kinetic energy there is a change in the intertial mass of a particle.

From the relativistic momentum, we have,

<u>p</u>=m.<u>u</u>
=m.<u>u</u>/(1-uý/cý)^«

Hence,

pý.cý+m$_o$ý.c^4=(m$_o$ý.uý.cý/(1-uý/cý))+m$_o$ý.c^4
=(m$_{oýuýcý}$+m$_o$ýc^4-m$_o$ýuýcý)/(1-uý/cý)
=m$_o$ýc^4/(1-uý/cý)
=Eý
Eý=pýcý+m$_o$ýc^4
E=c.(pý+m$_o$ýcý)^«

This relation is used frequently in high energy nuclear physics to calculate the total energy of a particle from it's momentum and vice versa.

<u>Example</u>
High energy particles are generally obtained by accelerating the particles in an accelerator. If a particle of charge q is accelerated throug a potential difference Vo, then the gain in kinetic energy is q.Vo. This give the following energy, 1.602 10^{19} C 1 V = 1.602 10^{9}= 1 eV. The **electron volt** is used as the standard unit of energy in particle physics.

Calculate the momentum of an electron of kinetic energy 2 Mev. We know that,

$$\mathbf{E^2=p^2c^2+m_o^2c^4}$$

$$\mathbf{pc=(E^2-m_o^2.c^4)^{1/2}}$$

The rest mass energy of an electron is,

$$\mathbf{m_o.c^2=9.11\ 10^{-31}\ Kg\ .\ (3\ 10^8 m/s)^2}$$
$$\mathbf{=8.20\ 10^{-14}\ J}$$
$$\mathbf{=0.511\ MeV}$$

We still need to work out the total energy however, but we do know the kinetic energy. So we write,

$$\mathbf{E=T+m_oc^2}$$
$$\mathbf{=2.000\ Mev + 0.5111\ MeV}$$
$$\mathbf{=2.5111\ MeV}$$

$$\mathbf{pc=((2.5111\ MeV)^2-(0.5111\ Mev)^2)^{1/2}}$$

$$\mathbf{=2.46\ MeV}$$

$$\mathbf{p=2.46\ MeV\ /\ c}$$

5.2 Invarient quantities in Special Relativity

A quantity which is unchanged by a general coordinate transformation is called an **invarient** of the transformation. In Newtonian mechanics invarients are for example,

1) Length, L=L'

2) Time T=T'

In special relativity invariants are;

1) $\mathbf{L^2-c^2.T^2 = L'^2-c^2T'^2 = S^2}$
   S is usually called the separation in "space-time"

2) $\mathbf{E^2-c^2p^2 = E'^2-c^2p'^2 = (m_oc^2)^2}$

5.3 Space-Time diagrams

In space time there are now four dimensions. We cannot percieve four dimensions, so we usually plot one spacial dimension with the time dimension. Lines on space-time diagrams are called **world lines**

Fig one
Fig two
Fig three

If Lý>cýTý
The separation S is a spacelike interval i.e., it is always possible to find a reference frame for which T'=0. This means it is at the same time but they have a seperation in space.

If Lý<cýTý
S is a timelike interval i.e., it is possible to find a frame for which L'=0. This means we have the same point in space but there is a temporal seperation.

Note:

Only timelike intervals, which are in the past and the future can be reached by radio signals which travel with the speed of light. Therefore, causality is preserved.

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6. Experimental verifacation of Relativity

6.1 Time dilation and Muon decay

Collisions of cosmic rays with atoms in the atmosphere at a hight of approximately 60 Km produce muons with velocity v=0.999 c . At that speed the muons would take a time T=x/v=60Km/0.999c=2 10^{4} seconds to reach the surface of the earth. The muons at rest in the laboratory have a half-life of approximatley 1.5 10^{6} s. According to a terrestrial observer, in 2 10^{4} s approximately 133 half-lievs would have passed. We only expect to observe «^{133}=10^{-40} of the origonal muons would reach the surface.

Experiment:
The number of muons at the surface of the earth (sea level) is much larger than would be expected. In the muons frame of reference the time T'=(1-vý/cý)«.T=9 10^{6} s, which is only 6 half-lives. i.e. «^{6}=1/64 of the origonal muons reach earth. This agrees with experiment.

6.2 Time dilation and atomic clocks.

Experiment:
Two atomic clocks, one clock is put onto a high speed plane travelling around the earth. The second (identical) clock remains in the laboratory. On the return of the plane, we find that it's atomic clock was moving slower.

6.3 The binding energy of the nucleus.

The total mass of a nucleus is less than the sum of the rest masses of the individual protons and neutrons making up the nucleus. The *mass defect* is defined as,

$$\mathbf{dm=Z.m_H+(A-Z).m_n-M_{Z,A}}$$

Where $M_{Z,A}$ is the atomic weight of an atom containing Z protons and A-Z neutrons. The mass defect is due to the binding energy of the nucleus.

$$\mathbf{dE=dm.c^2}$$

(Recall the plot of binding energy per mass number versus mass number. The maximum of this peak corresponds to a mass number of 56 (Fe, iron). Fusion relates to the lower side of the maximum moving towards the maximum and Fission to the higher side moving to the maximum.)

7. Paradoxes, general relativity and more.

7.1 Paradoxes

a) The Twin Paradox

Two twins, T and S. T stays at home and S goes on a space journey.
T says : S's clock runs slower so when S returns S will be younger on his return
S says : T's clock runs slower so T will be younger.
This is our Paradox. There is only an apparent symmetry, but special relativity is only okay for T bcause he does not travel (S will have to accelerate). Answer: S is younger than T.

10/12/98

7.2 The Theory of General Relativity

Special Relativity ;

- Applies to observers in uniform relative motion
- Incorporates electromagnetism (c=constant)
- Does not include gravitation.

It is that last point that lead Einstein to General Relativity (in 1916).

The Principal of equivalence
It is impossible to determine if an observer's reference frame is moving with accelerated motion or is in local gravitational field. Space-time is

determined my mass distribution. To a first approximation,

dx’=(1-((GM)/(cý.r)).dx

dt’=(1-((GM)/(cý.r))).dt

dx and dt are space/time intervals between two events where the gravitational potential is zero.
Therefore,

v’=(1-((2GM)/(cý.r))).v

Experimental Observations

1) Solar Eclipse of 1917. (**Fig One**)
People were able to see that stars due to the darkness. A star was observed which was apparenetly *in the wrong place* ! The change in the position of the star was explained by general relativity. The light from the star was deflected by the spatial distortion of the sun’s mass.

2)Perihelion presccesion of the planet Mercury.

3)Frequency of elecromagnetic radiation dependency on gravitational potential.

4)The Preccession of the orbits in binary star systems.

Predictions

Paths of light rays are strongly affected near massive commpact starts. The velocity of light , c’=0 occurs at, R_s=2GM/cý (R_s=Schvarzchild radius). A sphere with radius R_s is called an **event horizon**, because an observer external to the surface cannot recieve a signal produced inside. => **BLACK HOLE** !!!

17/12/98

Some problems

Q: A beam of energetic muons are projected into an evacuated tube 1Km long. It is found that only half of the muons reach the end of the tube, the rest decay in flight. Given that the half life of muons, i.e. the time in their rest frame after which half have decayed is 1.5 æs, calculated their velocity.

A: Time of travel is lab frame, t.. The proper half life is t'.=1.5æs. Given time dilation, t.=*y*.t'.. Velocity in lab frame, v=l/t.=l/(*y*.t'.),

vý=lý/(*y*ý.t.ý)
=(lý/t'.ý).(1-vý/cý)
vý/cý=(lý/cý.t.ý)/(1+(lý/(cýt'.ý))
=> v=0.91 . c

Q: A rocket moves past the earth at (3/5)c, at a later time the rocket moves past the earth at (3/5)c and at a later time sends back a scout rocket, which travels at a speed (4/5)c with respect to the parent rocket. With what speed does the scout rocket travel with respect to the earth ?

A: Assume frame S is fixed with respect to earth, and assume frame S' is fixed with respect to main rocket. We know that the velocity of the scout rocket in S' is

$$u_x'=-(4/5)c$$

We also know that the relative velocity between the frames is,

$$v=(3/5)c$$

The transformation of velocity equation gives us,

u_x=(u_x'+v)/(1+(u_x'v/cý))
=(-(4/5)c+(3/5)c)/(1-(12/25))
=-(5/13).c

Q: A beam of protons is incident on an hydrogen target. Calculated the minimum energy an incident proton must have in order for the following reaction to proceed.

$$p+p \rightarrow p+p+k^{+}+k^{-}$$

(proton rest mass = 1GeV.c^-2,
kaon rest mass = 0.5 GeV.c^-2)

A: Before collision, we have particle A (proton) with mass and velocity on target to hit particle B (H_2 nucleus) with the same mass, but at rest. After the collision we have two particle with mass m_p and two particles with mass m_k. At the threshold the particles produced have no kinetic energy. Apply the equation,

Mýc^4=Eý-pýcý

Consider, before the collision,

$$\textbf{Total } E=E_A+m_p c\text{ý}$$
$$\textbf{Total } P=P_A+\ldots$$

but for A,

$$m_p\text{ý}.c^4=E_A\text{ý}-P_A\text{ý}.c\text{ý}$$
$$P_A\text{ý}=(E_A\text{ý}/c\text{ý})-(m_p\text{ý}c\text{ý})$$

Going back to the previous equation,

$$M\text{ý}c^4=(E_A+m_pc\text{ý})\text{ý}-E_A\text{ý}+m_p\text{ý}c^4$$
$$M\text{ý}c^4=2E_A.m_pc\text{ý}+2m_p\text{ý}c^4 \qquad (1)$$

After

$$\textbf{Total } E=2m_pc\text{ý}+2m_kc\text{ý}$$
Total P=0 (by defintion)

$$M\text{ý}c^4=(2m_p+2m_k)\text{ý}.c^4 \qquad (2)$$

From (1) and (2),

$$E_A=((2m_p+2m_k)\text{ý}.c^4-2m_p\text{ý}.c^4)/(2m_pc\text{ý})$$
$$E_A=((2\text{ GeV}+1\text{ GeV})\text{ý}-(2\text{Gev})\text{ý})/(2\text{ GeV})$$
$$E_A=3.5\text{ GeV}$$