

Communications

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I Introduction to Terminology

Basics ideas and the need for maths, foundation of the principals to allow communication clearly over distances. The function of a communication system is to transfer information from one point to another point via some communication link (or a "**channel**").

If you have a microphone linked to a loud speaker, the wire connecting the two is the channel. The microphone is a **transducer** It converts energy types. There is also energy losses accompanying transmission sources, this is known as **NOISE**. The received signal is associated with a random, erratic, voltage waveform. When the noise is totally random, it is called **white noise** **Pink noise** is however, random but confined. It is usually used for diagnostic purposes. The message signal voltage may not be large in comparison with noise voltages.

One of the principal concerns of communication theory is to suppress as far as possible the effects of noise. It is better not to transmit the original signal, but use this signal to generate a different waveform, which is then transmitted.

This process is called, **Encoding** or **Modulation** The reverse is **decoding** or **Demodulation** respectively.

Simultaneous Transmission over a channel of more than one waveform is called **Multiplexing**.

We will be dealing with many different waveforms. The branch of maths used for this is Spectral analysis. We deal with the **frequency domain** for this.

The Channel

Everything that intervenes between the original signal and the final recovered signal.

Channel Capacity

The maximum rate of information that can be passed over the channel.

We want the channel to transmit digital information. This is a sequence, in time, of BITS. Logical 0's and 1's. Thus, in successive intervals we may want to transmit one of two possible messages.

Message, M_0 that a bit 0 is intended, or M_1 that a bit 1 is intended. In the end, the two possible messages might be represented at the transmitting end by two distinct waveforms. Each limited in time duration to the interval allocated to a bit. At the receiving end we might devise a system where by the message M_0 when received generates some voltage r_0 . M_1 received generates a voltage r_1 .

In the absence of noise, message M_0 generates r_0 and M_1 generates r_1 with complete certainty. You may with noise send M_0 but r_1 is generated and vice versa. We need to develop algorithms which will serve to allow us an opinion about the message with the maximum probability that our opinion is correct. We have to study probability theory and it's relation to information theory.

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II Amplitude Modulation Systems

A means by which multiplexing may be achieved consists of translating each message to a different position in the frequency spectrum (this is frequency multiplexing). The individual message can be separated from the rest by means of filtering. Frequency multiplexing uses an auxiliary wave form, usually sinusoidal, this is called the **Carrier**, (in old terms, *Carrier wave*). The resulting modified carrier wave is called a **Modulated Carrier**. In some cases the modulation is related simply to the message, however the relationship is quite complicated.

Frequency Translation

Often advantageous and convenient to translate the signal from one region to another in the frequency spectrum. It can aid processing. Say the original signal lies between f_1 and f_2 , we can then translate that to f_1' and f_2' .

An audio tone of 1KHz, $\lambda=300,000$ m

Antenna-radiate and receive (E.M.) signals. They operate only when their dimensions are approximately equal to the wavelength (λ) of the signal received.

For example, a band of 50 Hz to 10 KHz. The ratio to the highest to the lowest is 200. We would need an antenna that changes it's own dimension's by a factor of 200!

If we translate $(10^6 + 50)$ Hz to $(10^6 + 10^4)$ Hz. The ratio is now 0.01! This turns a "Wide Band" signal to a "Narrow Band" signal. We have done "Narrow Banding". These denote fractional changes in the frequency from one band edge to another.

A method of Frequency Translation:

Multiply the signal with an auxiliary sinusoidal signal.

$$\begin{aligned} &\textbf{Initial signal,} \\ &\mathbf{v_m(t)=A_m \cdot \cos(\omega_m t)} \end{aligned}$$

$$\begin{aligned} &\textbf{Auxiliary signal,} \\ &\mathbf{v_c(t)=A_c \cdot \cos(\omega_c t)} \end{aligned}$$

$$\begin{aligned} &\mathbf{v_m(t) \cdot v_c(t)=} \\ &\mathbf{=(A_m \cdot A_c / 2) [\cos((\omega_c + \omega_m)t) + \cos((\omega_c - \omega_m)t)} \end{aligned}$$

We have two distinct wave forms, one of frequency $f_c + f_m$ and one of $f_c - f_m$

$v_m(t)$ - Spectral range, this is the BASE BAND FREQUENCY RANGE.
 $v_m(t)$ is the base band signal.

Operation of multiplying a signal with an auxiliary signal is called mixing or Heterodyning.

In the translated signal, the part which consists of spectral components above the auxiliary signal in the range $f_c + f_m$, this is the Upper sideband signal. Those from $f_c - f_m$ are the Lower sideband signal.

$f_c + f_m$ is the **sum frequency**

$f_c - f_m$ is the **difference frequency**

Auxiliary signal of frequency f_c is known as the local oscillator signal, mixing signal, Heterodyning signal or as the carrier signal depending on the type of application.

Note: The process of translation results in a signal that occupies the range $f_c - f_m$ to $f_c + f_m$. There are other translation methods, but this is the simplest.

Recovery of the Base Band signal

Again, simply multiply the translated signal with **$\cos(\omega_c t)$** .

$$\mathbf{m(t)A_c \cos(\omega_c t) \cos(\omega_c t) = m(t) \cdot (A_c / 2) \cdot [1 + \cos(2\omega_c t)]}$$

Removes base band by filtering ($m(t)$ is the translated carrier signal).

Note: In addition to the base band there is a signal whose spectral range extends from $2f_c$ to $2f_c + f_m$. This can cause difficulties since $f_c - f_m$. Spectral range of the double frequency signal is widely separated from the base band. Hence, $2f_c$ signal is easily removed by a low pass filter.

Amplitude Modulation

A frequency translated signal from which the base band signal is easily recoverable is generated by adding to the product of the base band and carrier, the carrier itself.

$$v(t) = A_c [1 + m(t)] \cdot \cos(\omega_c t)$$

The resultant waveform is one in which the carrier, $A_c \cdot \cos(\omega_c t)$ is Modulated in amplitude. The process of generating such a waveform is called Amplitude modulation (AM for short).

Problem 1

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The great merit of an AM carrier is the ease by which the base band signal is recovered.

fig 1

(4 terminal circuit with R & C in parallel across Output. Diode is used on top input line to restrict current flowing input.)

Assume a fixed amplitude and initially R is not present. C charges to the peak positive voltage of the carrier. Suppose input carrier amplitude is increased, the capacitor charges to the new higher value. Capacitor will hold this voltage as the diode will not conduct in the reverse direction. In order for the capacitor voltage to follow the carrier wave amplitudes when they are decreasing it is necessary to include R so that C, the capacitor may discharge (to earth).

Make the time constant RC so that the change in V_c between cycles is at least equal to the decrease in carrier amplitude between cycles.

fig 2

Phase (of Frequency) Modulation

AM systems: Modulator output which consists of a carrier which displays variation in it's amplitude.

FM systems: System where the modulator output is of a constant amplitude and the signal variations are superimposed on the carrier through variations in the carrier frequency.

AM system:

Each spectral component of the base band signal gives rise to

one or more spectral components in the modulated signal frequencies of the spectra components do not depend on the amplitudes of the input signals only on the frequencies of the carrier and base band.

All operations performed on the signal are linear operations, so the (rule of) supposition applies.

i.e.

$M_1(t)$ - 1st spectrum

$M_2(t)$ - 2nd spectrum

SUM $M_1(t)+M_2(t)$ will introduce a spectrum which is the sum of the separate spectrum components.

New type of modulation - spectral components in the modulated wave form depend on the amplitude as well as the frequency of the spectral components in the base band signal. It is non linear, and as such, supposition does not apply.

The form of the signal is going to be,

$$v(t) = A \cdot \cos(\omega_c t + \theta(t))$$

$\omega_c = \text{constant}$, $\theta(t)$ is a function of the base band signal.

Angle Modulation or Phase Modulation

(Frequency modulation)

Review of angular frequencies

$$A \cdot \cos(\omega_c t) = \text{Re}(A \cdot e^{j\omega_c t})$$

$A \cdot e^{j\omega_c t}$ is in the complex plane, it is a PHASOR of length A and angle θ (at a rate of ω_c).

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If $\theta = \omega_c t$ it rotates counter-clockwise with angular frequency ω_c . If you have phase which changes with time, $\theta = \theta(t)$ then $V(t)$ would be represented by a phasor of amplitude A which runs ahead or falls behind the phasor representing $A \cdot \cos(\omega_c t)$. If phasor of angle $\theta + \theta(t) = \omega_c t + \theta(t)$ it alternately runs ahead or falls behind the phasor of $\omega_c t$. Then the first phasor must be alternately be rotating more or less rapidly than the second phasor. i.e. the angular velocity of the phasor $V(t)$ undergoes a modulation around the nominal angular velocity ω_c . The angular velocity associated with the argument of a sinusoidal function is equal to the time rate of change of the argument (argument being angle), i.e. the cycle of the function.

For instance the instantaneous radial frequency of w is $d(\theta)/dt$ corresponding frequency $(w/2\pi)$ is,

$$\dot{\theta} = \frac{1}{2\pi} \frac{d}{dt}(\omega t + \theta) = \frac{\omega}{2\pi} + \frac{1}{2\pi} \left(\frac{d\theta}{dt} \right)$$

Hence the waveform $V(t)$ is modulated in frequency initially the waveform having fixed frequency and phase. If the frequency variation is small, about ω_c i.e. $d\theta/dt$ is \ll than ω then the resultant waveform is recognisable as a sine wave, i.e. the period or change from cycle to cycle.

$\dot{\theta}$ is the instantaneous frequency $\dot{\theta}_c$ is the frequency of the carrier and $\theta(t)$ is the instantaneous phase.

Design of the Modulator

- 1) $\theta(t)$ is directly proportional to the modulating signal
or
 - 2) $\theta(t)$ is proportional between the modulating signal and the $d\theta/dt$ with $\dot{\theta}_c = \omega_c/2\pi$ and $d\theta/dt = 2\pi(\dot{\theta} - \dot{\theta}_c)$.
- 1 is phase modulation and 2 is frequency modulation.

Relationship between phase and frequency modulation

Output, $V(t)$, which is a carrier phase modulated by input signal $m_i(t)$.

$$V(t) = A \cos(\omega t + k' m(t))$$

k' is a constant and $m_i(t)$ may be derived from the integral of the modulating signal $m(t)$ so that,

$$m_i(t) = k'' \cdot \int_{-\infty}^t m(t) dt$$

Take that,

$$k = k' k''$$

then,

$$V(t) = A \cos[\omega t + k \cdot \int_{-\infty}^t m(t) dt]$$

Hence instantaneous angular frequency,

$$\begin{aligned} \omega &= \frac{d}{dt} (\omega t + k \cdot \int_{-\infty}^t m(t) dt) \\ &= \omega_c + k \cdot m(t) \end{aligned}$$

The deviation of instantaneous frequency from the carrier frequency

$\omega_c/2\pi$, is $\phi = \int \dot{\phi} dt = (k/2\pi)m(t)$.

The technical derivation of instantaneous frequency being proportional to the modulating signal is by a combination of integrator and phase modulation devices.

If $\phi(t)$ is proportional to $m(t)$ we have phase modulation. If $d\phi(t)/dt$ is proportional to $m(t)$ we have frequency modulation.

Phase deviation: maximum phase deviation of the total angle from the carrier angle $\omega_c t$.

Frequency deviation: is the maximum departure of instantaneous frequency from the carrier frequency.

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Angular variation (& consequently frequency). Variation is sinusoidal with frequency, f_m . $\omega_m = 2\pi f_m$.

$$v(t) = A \cos(\omega_c t + \hat{a} \sin(\omega_m t))$$

\hat{a} = maximum peak amplitude of $\phi(t)$ (= maximum phase deviation). It is also called the **MODULATION INDEX**.

Instantaneous frequency,

$$f = \omega_c / 2\pi + \hat{a} (\omega_m / 2\pi) \cos(\omega_m t) \\ = f_c + \hat{a} f_m \cos(\omega_m t)$$

Maximum frequency deviation, $df = \hat{a} f_m$.

Therefore,

$$v(t) = a \cos\{\omega_c t + (df/f_m) \sin(\omega_m t)\}$$

f range: $f_c \pm df$.

Spectrum of an FM signal

$$v(t) = \cos(\omega_c t + \hat{a} \sin(\omega_m t)) \\ (A=1)$$

$$\cos(\omega_c t + \hat{a} \sin(\omega_m t)) = \cos(\omega_c t) \cos(\hat{a} \sin(\omega_m t)) - \sin(\omega_c t) \sin(\hat{a} \sin(\omega_m t))$$

Consider,

$$\cos(\hat{a} \sin(\omega_m t))$$

this is an even function.

Angular frequency ω_m expands this as a Fourier series with $\omega_m/2\pi$ as a fundamental. The result is that there is no evaluation of the odd coefficients, (which are functions of \hat{a}) - ie, even functions are present only (since cosine is even itself), odd functions/harmonics are zero.

$$\cos(\hat{a}\sin(\omega_m t)) = J_0(\hat{a}) + 2J_2(\hat{a})\cos(2\omega_m t) + 2J_4(\hat{a})\cos(4\omega_m t) + \dots + 2J_n(\hat{a})\cos(n\omega_m t) + \dots$$

Now consider the odd function (this only contains odd harmonics),

$$\sin(\hat{a}\sin(\omega_m t)) = J_1(\hat{a}) + 2J_3(\hat{a})\sin(3\omega_m t) + 2J_5(\hat{a})\sin(5\omega_m t) + \dots + 2J_{n-1}(\hat{a})\sin((n-1)\omega_m t) + \dots$$

$J_n(\hat{a})$ is a **Bessel function** of the first kind with order n .

Putting these results back into $v(t)$,

$$v(t) = J_0(\hat{a})\cos(\omega_c t) - J_1(\hat{a})[\cos((\omega_c - \omega_m)t) - \cos((\omega_c + \omega_m)t)] + J_2(\hat{a})[\cos((\omega_c - 2\omega_m)t) + \cos((\omega_c + 2\omega_m)t)] - J_3(\hat{a})[\cos((\omega_c - 3\omega_m)t) - \cos((\omega_c + 3\omega_m)t)]$$

Spectrum : Carrier with amplitude $J_0(\hat{a})$ and a set of side bands spaced symmetrically on either side of the carrier at frequency separations $\omega_m, 2\omega_m$, etc. This system is non-linear.

see sheet (marked "figure 5" and "figure 6")

Now, if $\hat{a}=0$, $J_0(0)=1$ and $J_n=0$. No modulation occurs, only the carrier of normalised amplitude unity is present.

If, $\hat{a} \ll 1$ $J_0(\hat{a}) = 1 - (\hat{a}/2)^2$ (approx)

and, $J_n(\hat{a}) = (1/n!) \cdot (\hat{a}/2)^n$ (approx) $n > 0$

If \hat{a} is very small, the FM signal is composed of a carrier and a single pair of side bands with frequency $\omega_c \pm \omega_m$. Narrow band FM signal.

As \hat{a} increases J_n 's become more significant.

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Band Width of a sinusoidal modulated FM signal

For an FM modulated signal the number of side bands can be infinite. Hence the bandwidth to encompass such a signal must also tend to infinity. In practice for any large \hat{a} a fraction of the power in the signal is confined to the side bands which lie within a finite band width such that no serious distortions occur. $J_0(\hat{a})$ and $J_n(\hat{a})$ hugs the zero axis initially as n increases J_n remains close to the zero axis upto quite large values of \hat{a} . We only need to consider the J_n 's which have succeeded in making a significant departure from the zero axis (figure

6).

Experimentally, bandlimiting an FM signal to 98% or more power is passed by filter limiting. This gives tolerable distortion. However, remember the FM signal, the amplitude of the spectral component at f_c is not constant and is independent of β . This means that, also the envelope of an FM signal has a constant amplitude so that the power of such a signal is a constant and is independent of the modulation. Hence,

Power a Ampy

The power of a unit amplitude signal $P_v = 1$ and is independent of β .

$$J_0^2 + 2J_1^2 + 2J_2^2 + \dots = P_v$$

We can calculate P_v by squaring $v(t)$ and averaging $v^2(t)$. This is actually independent of β .

$$P_v = \{ J_0^2 + \sum_{n=1}^{\infty} 2J_n^2 \}$$

note. sum value of $J_0(\beta) = 0$ i.e. all the power is in the side bands.

$$\begin{aligned} P &= \{ J_0^2(1) + J_1^2(1) + J_2^2(1) \\ &= 0.289 + 0.193 + 0.013 \\ &= 0.495 \\ &\sim 99\% \text{ of } 0.5 \end{aligned}$$

Lines occur always after $n = \beta + 1$ thus the bandwidth required to transmit and received signal sinusoidally modulated FM signal is,

$$\begin{aligned} B &= 2 \cdot (\beta + 1) \cdot f_m \\ &\text{or} \\ B &= 2 \cdot (df + f_m) \quad (df = \beta \cdot f_m) \end{aligned}$$

"The bandwidth is twice the sum of the maximum frequency deviation and the modulating frequency". This is **Carson's Rule**

Information Theory

Information is the difference between knowing and not knowing something. Alternatively, being faced with a number of possibilities and between knowing the one which actually prevails.

Example

We have a choice between n possibilities. Say, an object is hidden in n boxes. The result is mutually exclusive because there is only one choice. Also the choice of the box equally probable. We have an

inability to decide, we lack information. When the information i is supplied one possibility is chosen. We can say,

$$i=i(n)$$

However, if you have $n=1$ then $i(n)=0$, there is no need for any information. As n goes to infinity then the information missing goes to infinity as well. If you have n and m choices and $n>m$ then the information required for n choices is greater than for m choices. Consider now the problem between two independent problems of choice one has n possibilities and the other has m possibilities. So the total number of possibilities is $n*m$. i can be supplied in two steps, information about $i(n)$ and then information about $i(m)$. Which is the same as informing about $i(m)$ and $i(n)$ together. We can say that the information can be split

$$i(nm)=i(n)+i(m)$$

$$i(n/m)=i(n)-i(m)$$

An expression which will satisfy these requirements is

$$\ln(nm)=\ln(n)+\ln(m)$$

We have missing information when a probability distribution is given.

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If there is a choice of n possibilities, now assigned a probability to each possibility.

$$p_i > 0 \quad i=0 \dots n$$

$$\sum_{(i=1, n)} p = 1$$

Reduce the case to equal probability possibilities. The frequency of answers should become proportional to the possibilities as the number of problems go to infinity. Replace n possibilities with N independent problems.

As N reaches infinity, the possibility i is correct in $N \cdot P_i$ cases.

Now, which of the N problems will be the $N \cdot P_i$ with the answer i .

All orders are equally probable, such that the total number is

$$N! / \text{SumProd}_{(i=1, n)} \{(N_i) P\}$$

Missing information for N problems such that,

$$I_N = k \cdot \ln(N! / \text{SumProd}_{(i=1, n)} \{(N_i) P\})$$

$$=k \cdot [\ln(N!) - \sum_{i=1, \dots, n} \{\ln(N!)\}]$$

I_N as N goes to infinity $\ln(N!) = N \cdot \ln(N) - N + O(1)$,

$$I = (\lim_{N \rightarrow \infty} N) \cdot \sum_{k=1}^N \{ \ln(P_k) \}$$

Amount of Information

If we have a communication system in which the allowable messages are m_1, m_2, \dots with a probability of occurrence P_1, P_2, \dots . Since the sum of all probabilities always equals one, the transmitter selects a message M_k or probability P_k . You assume the receiver correctly identifies the message. A definition of the term *information* that the system has conveyed an amount of information.

$$I_k = -\log(P_k)$$

Recall that,

$$\begin{aligned} \log_2(N) = x, \log_e(N) = y, N = 2^x, N = e^y \\ \text{Therefore,} \\ 2^x = e^y, x = y \cdot \log_2(e) \\ \text{i.e. } \log_2(N) = (\log_2(e))(\log_e(N)) = \text{Const.} \cdot \ln(N) \end{aligned}$$

I_k is dimensionless, by convention it is assigned a unit called the bit. M equally likely and independent messages and that $M = 2^5$

$$I = \log_2(M) = \log_2(2^5) = 5 \text{ bits.}$$

Average Information

M_1, M_2, \dots with P_1, P_2, \dots . During a long sequence, L messages have been generated. Then if L is large then in the L message sequence we have transmitted $P_1 \cdot L$ messages of M_1 , $L \cdot P_2$ of M_2 , and so on.

$$I_{\text{TOTAL}} = P_1 \cdot L \cdot \log_2(P_1) - P_2 \cdot L \cdot \log_2(P_2) \dots$$

Available information per message interval.

$$H = I_{\text{TOTAL}} / L = -\sum_{i=1, \dots, n} \{ P_i \log_2(P_i) \} \text{ (in bits)}$$

This is the ENTROPY of the message

H for an extremely likely message $= 0$,

H for an extremely unlikely message $= 0$

Example,

2 messages ie $n=2$,

$$H = -P_1 \cdot \log_2(P_1) - P_2 \cdot \log_2(P_2)$$

$$H = -\text{const.} \cdot (P_1 \cdot \ln(P_1) + (1 - P_1) \cdot \ln(1 - P_1))$$

Find the maximum dH/dP_1

$dH/dP_1 = \text{const.}(\ln(P_1) - \ln(1-P_1))$, i.e. $\ln(P_1/(1-P_1))=0$, or $P_1/(1-P_1)=1$ which implies $P_1=1/2$. H is a maximum when $P_i=1/M$. $H_{\max} = \log_2(M)$.
 $P=1/M$ which implies

$$\text{Sum (1 to M) } \{(1/M) \cdot \log_2(M)\}$$

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M messages proven that H is a maximum when messages are equally likely. H_{\max} occurs when $P_i=1/M$. Maximum information/messages = ...

$$\dots - \text{Sum (i=1,M) } (\log_2(P_i)) \\ = \text{Sum (i=1,M) } ((1/M) \cdot \log_2(M))$$

Channel Capacity

All practical communication channels are affected by noise or distortion. We need to be able to predict how much information can be passed over a given channel with specified physical characteristics. i.e. its capacity. There is no such thing, in a strict sense, as a digital channel since noise will introduce uncertainty in levels, i.e. continuum of values around nominal discrete levels.

Sampling Theorem

We need to sample an analogue signal then later reconstruct the original signal which comes from the samples. Sampling must take place at a certain minimum rate. This is known **Nyquist Theorem**. This theorem states that the sampling rate must equal twice the base band frequency, otherwise an effect known as **aliasing**.

a) Signal with a low pass spectrum. We know that the minimum frequency for sampling is $f_s \geq 2 \cdot f_c$. Sampling results in a periodic spectrum.

Fig One
Fig Two

If $f_s < 2 \cdot f_c$, the components spectrum will overlap. This overlap leads to ambiguity (aliasing) and the signal will not be passed by a low pass filter.

b) Signal with a bandpass spectrum,

Minimum sampling rate, $f_s \Rightarrow f_h - f_L$. i.e. twice the bandwidth.

Fig. Three

Derivation of channel capacity

Consider a channel that is wanted with a peak amplitude $\bar{n}A$ volts corrupted additive noise with peak amplitude $\bar{n}B$ volts.

Fig. Four

Signal levels are just distinguishable when levels are separated by $2B$ volts.

Total number of separable signal levels = Total amplitude range (signal + noise) / Minimum level separation.

$$= (2A + B_{\text{TOP}} + B_{\text{BOTTOM}}) / 2B \\ = (A+B) / B$$

Signal and Noise are uncorrelated, they are independent of each other. Also, they are time independent. I.e. no long term similarity. The time average of $[A+B]\hat{y} = \langle (A+B)\hat{y} \rangle$.

$$[A+B]\hat{y} = \langle (A+B)\hat{y} \rangle = \langle A\hat{y} + 2AB + B\hat{y} \rangle \\ = \langle A\hat{y} \rangle + \langle B\hat{y} \rangle \\ \text{this goes to zero.}$$

$$\langle A\hat{y} \rangle \text{ a signal power (S)} \\ \langle B\hat{y} \rangle \text{ a noise power (N)}$$

From (1) ,

$$(A+B)/B = ((S+N)/N)^{\llcorner}$$

This gives the number of distinguishable signal levels. We assume that all levels are equally probable, then the information associated with amplitude level,

$$I = \text{Log}(Q) \\ Q = \text{No. of states} \\ I = \text{Log}((S+N)/N)^{\llcorner} \\ = \llcorner \text{Log}((1+S/N)^{\llcorner}$$

bits per independent sample of the noisy signal.

$$c = (\text{No of bits per sample}) * (\text{no of samples / second}) \\ = \llcorner \text{Log}(1+S/N) * 2W \\ = W * \text{Log}(1+S/N) \text{ Bits/Sec}$$

Where W = bandwidth.

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The above expression for channel capacity is the **SHANNON-HARTLEY THEOREM FOR CHANNEL CAPACITY C**.

The source of messages generates messages at a rate of r messages/sec. The information rate $R=rH$ =average number of bits of information/sec.

Example:

An analogue signal band limited to B Hz sampled at Nyquist frequency and the samples are quantised into 4 levels. The quantisation levels (messages) $Q_1 \rightarrow Q_4$ are presumed independent and occur with possibilities $P_1=P_4=1/8$ and $P_2=P_3=3/8$. The average information $H=-\sum P_i \cdot \log_2(P_i) = 1.8$ Bits/Sec. The information rate is $R=rH=2B(1.8)=3.6*B$ Bits/Sec.

Shannon's theorem: It is possible to transmit information with an arbitrarily small probability of error provided that the information rate, R , is less than or equal to C , the channel capacity.

The approach to achieve this is coding. Given a source of M equally likely messages, with $M \geq 1$. Which is generating information at a rate of R . Given a channel with a channel capacity, c , then if $R \leq C$ there exists a coding technique such that the output of the source may be transmitted over the channel with a probability of error in the received signal which can be made arbitrarily small.

If $R \leq c$ transmission may be accomplished without error in the presence of noise. If $R > c$ the probability of error closes to unity.

c is simply a function of bandwidth and signal to noise ratio. It is possible to trade off between these two parameters. So if you have a bad S to N ratio you could increase your bandwidth.

Measurements and Measurement systems

The commonality is that measurement can be viewed as an attempt to extract information for or about a given process. Therefore, subject to the results of information theory. In particular,

$$c = W \cdot \log[1 + S/N] \quad (\text{Bits/Sec})$$

Limitations on Measurement Techniques.

The process of making a measurement is viewed as information transfer. A certain amount of information must be obtained by the

measurement system in order to specify a given physical parameter to a required degree of accuracy. Maximum amount of information which can be passed by the system without error is given by,

$$I_{\max} = WT \cdot \log_2(1 + S/N) \text{ BITS}$$

$$= cT$$

= c * (Interval over which the information is obtained)

Information is related to the predictability or randomness of parameter changes.

Now let us consider a frequency source that can take any value of frequency of upto 1 GHz that is required to measure the value of frequency to an accuracy of ± 1 Hz. How much information is necessary to achieve this? I is also defined by,

$$I = \log_2(\text{number of distinguishable bit states})$$

$$= \log_2(10^9) \text{ BITS}$$

The measurement system has a bandwidth, $w = 100$ KHz and $S/N = 20$ dB. The minimum time taken to make the measurement to the required accuracy,

$$T = I / (W \cdot \log_2(1 + S/N)) = 29.9 / (100 \cdot \log_2(101)) = 45 \text{ } \mu\text{s}$$

If we reduce the S/N ratio to 6 dB, $T = 129 \text{ } \mu\text{s}$.

If we now extend the example to where the source operates in a frequency hopping mode, where the hopping interval is 10ms. To maintain the accuracy (!),

$$L = I/T = w \cdot \log_2(1 + S/N) \text{ Bits/Sec}$$

$$I = 29.9 / 10\text{ms}$$

i.e. the information rate is 2990 bits/sec

T must not exceed 10ms but W and S/N can be varied. If c is the maximum rate of information transfer, take $S/N = 10\text{dB}$

$$c = 2990 = W \cdot \log_2(11)$$

Therefore,

$$w = 2990 / \log_2(11) = 864 \text{ Hz}$$

15/12/98

Recall, "optimum condition - when process and measurement bandwidth are equal. Measured bandwidth < process bandwidth we get some distortion"

Noise Sources

Let us consider communication systems. There are two classes of noise that need to be considered for such systems.

- Internal Noise
 - ↳ Noise generated by the thermal agitation of electrons in a conductor.
 - ↳ Noise due to statistical fluctuations in the number of electrons contributing to current flow, eg, in semiconductors.
- External Noise
 - ↳ Ignition interference.
 - ↳ Mains hum
 - ↳ Fluorescent lights
 - ↳ "static"
 - ↳ Switch contacts
 - ↳ Interference from other transmissions.

In principle external noise can be reduced or eliminated by improved screening, filtering and earthing, etc. EMC, "Electromagnetic Compatibility".

Internal noise: This forms a fundamental limitation on system performance. It cannot be eliminated. Thermal noise can be reduced by cooling.

Units: When dealing with different types and numbers of noise sources, it is convenient to treat different sources as being independent (ie, uncorrelated). This means that the long term average of the product of two noise waveforms goes to zero. We usually have mean square voltages and current (proportional to power) are used (usually associated with a 1 ohm load).

Noise type 1: Thermal noise (Johnson noise).
We know that this is due to the thermal agitation of electrons at temperatures greater than absolute zero, in a conducting material. Consider a 'noisy' resistance R connected across a band pass filter (bandwidth, w).

fig one

Experimentally, the maximum available average noise power,

$$P_{av} = kT\omega$$

If we break this down,

$$\text{Power} = \text{Energy} / \text{Time} = \text{Energy} * \text{Freq.}$$

Thermal energy $\propto kT$

**Freq. α Bandwidth
Power= kTW Watts**

All temperatures are from the absolute scale, in Kelvins.

Power is usable when connected to an external circuit.

Fig Two

Consider an alternating voltage (with RMS value) in series with a noise free resistance R , connected across a noise free load R_L . Maximum power is dissipated when $R=R_L$. Therefore, maximum power in the load is,

$$= (\langle v_o^2 \rangle / (2R)) \cdot R = \langle v_o^2 \rangle / 4R = kTW$$

Therefore,
 $\langle v_o^2 \rangle = 4kTWR$

$\langle v_o^2 \rangle$ open circuit means square noise voltage produced by a noisy resistor. We can design an equivalent system that has current representations of R .

Fig Three

Complex Impedances:

Recall, $Z=A+jB$ and $\langle v_o^2 \rangle = 4kTAW$. If A is a function of frequency and B is a function of frequency the W is a small band centered on f . If we plot the noise amplitude, the noise is greatest in the region of resonance.

Noise type 2: Shot Noise

Current carriers act as discrete charge transferring particles, rather than homogeneous current with uniform velocity. So the noise arises due to statistical fluctuations in electrons in material. We find that,

$$\langle i_o^2 \rangle = 2I_e e W$$

Where I_e is the direct emitter to collector current. e is the electron charge and W is the bandwidth. If we break this down,

$$\begin{aligned} i_o^2 &= \text{current} * \text{current} \\ &= I_e * \text{current} \\ &= I_e * (\text{charge/time}) \\ &= I_e * e * W \\ &\text{(2W is the Nyquist requirement)} \end{aligned}$$

Noise type 3: $1/f$ noise: In this the noise level is proportional to the reciprocal of the frequency.

THE END !
thanks for reading!