

SEMICONDUCTOR QUANTUM WELL PHYSICS

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Overview

We are mainly concerned with (in this course) solids. In particular what happens when we reduce the dimensions of the structures. The physics changes dramatically as we change the number of dimensions of a solid. The dimensions can either be macroscopic (1 cm (10^8 Å) for example) or microscopic (10 Å). A **3d** macroscopic solid means that it is 1cm*1cm*1cm. This is referred to as a **bulk material**. A **2d** is classed as 1cm*1cm*100Å. Hence a **1d** solid is classed as 1cm*100Å*100Å, and finally **0d** is classed as 100Å*100Å*100Å. Remember though, dimensions here are referring to **macroscopic** dimensions.

A 2d structure is called a **quantum well**. A 1d structure is called a **quantum wire**. A 0d structure is called a **quantum dot**.

Quantum dot lasers have been built with dimensions in the order (10Å)³ and an output of 1 micro-watt. If they are all stacked together into a structure of (1cm)³. The power of this structure is $10^{21-6}=10^{15}$ W, which is a large amount of power.

"It is win all the way with lowered dimensions!"

3D Solids

Say we have two Hydrogen atoms separated by infinity.

fig 1

Both atoms have their own (equal) energy level. If they are brought together a Hydrogen molecule is formed, this molecule has two energy levels, one is higher than the atomic energy level and the other is lower than the atomic energy. The lower is called the bonding level and the higher is called the antibonding level. This is typical energy level splitting. If a structure of 1 mole of atoms is brought together there are roughly 10^{23} energy levels together, the difference between these levels are tiny, the levels are called an **energy band**. When the gap between levels is so small we refer to it as a **quasi-continuous energy band**. An electron with energy **kT** at room temperature can easily move around quasi-continuous energy states/bands.

We need to know what the relationship is between the energy and the momentum of charge carriers such as electrons. These are two quantities which are always conserved. Recall that a particle with

momentum P has a De Broglie wavelength λ associated with it.

$$\begin{aligned} P &= h/\lambda \\ P &= (h/\lambda) \cdot (2\pi/2\pi) \\ P &= 2\pi \cdot \hbar/\lambda \\ &= \hbar \cdot k \\ (k &= 2\pi/\lambda) \end{aligned}$$

So if we want to connect the energy and the momentum of the particle we can equally say we want to connect the energy and the wave vector, k .

It has taken theoreticians a long time to solve this from first principals, experimentalists solved it earlier with a slightly flawed method.

Suppose an electron is moving around in free space, it has no potential energy, so all of its energy is kinetic (it is also moving around at non-relativistic speeds). We can express the energy as,

$$\begin{aligned} E &= \frac{1}{2}mv^2 \\ E &= P^2/2m \\ E &= (\hbar^2/2m)k^2 \end{aligned}$$

In a vector relation we can say,

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

Therefore for the electron moving around in free space,

$$E = (\hbar^2/2m) \cdot (k_x^2 + k_y^2 + k_z^2)$$

Just consider,

$$\begin{aligned} \partial E / \partial k &= (\hbar^2/m) \cdot k \\ \partial^2 E / \partial k^2 &= \hbar^2/m = \partial^2 E / \partial k_x^2 = \partial^2 E / \partial k_y^2 \\ \partial^2 E / \partial k_x \partial k_y &= 0 = \partial^2 E / \partial k_y \partial k_x = \partial^2 E / \partial k_x \partial k_z \end{aligned}$$

(We will see the significance of this later.)

We can generally say,

$$E = a(k_x^2 + k_y^2 + k_z^2)$$

Experimentalists made a guess at the solution that the above can be expanded as a power series in the wave vector. However, the series could go on forever and it is unknown how far to go in the series.

$$E = a_0 + a_1 k_x + a_2 k_y + a_3 k_z + a_4 k_x^2 + a_5 k_y^2 + a_6 k_z^2 + a_7 k_x k_y + a_8 k_x k_z + a_9 k_y k_z$$

It is known in principle why this seems okay. The momentum operator commutes with the Hamiltonian which means the

momentum components are *good quantum numbers* (they are conserved and constant). This means that one good quantum number can be expanded with other good quantum numbers. The only problem is where to truncate the series.

The argument which led to the power series solution can be simplified by symmetry arguments.

Symmetry Arguments

Consider a cubic crystal of Zinc Sulphide, with a zinc atom in the centre and eight sulphur atoms around it forming a cube. This is the unit cell of the macroscopic crystal, this is called point group symmetry. The point group is a set of operations that sends the unit cell into itself. If the cell is observed before and after such an operation no difference is seen, the energy remains the same.

Now let us consider a rotation by $\pi/2$.

fig 2

If we rotate the orthonormal set of x,y and z, we see that for a rotation of $\pi/2$,

- z goes into x
- x goes into y
- y goes into -x

Therefore applying this argument to the wave vector of the atom and electrons in the crystal,

k_z goes into k_x

k_x goes into k_y

k_y goes into $-k_x$

Hence we write the power series as,

$$\mathbf{E} = \mathbf{a}_0 + \mathbf{a}_1 \mathbf{k}_y + \mathbf{a}_2 (-\mathbf{k}_x) + \mathbf{a}_3 k_z + \mathbf{a}_4 (\mathbf{k}_y)^2 + \mathbf{a}_5 (-\mathbf{k}_x)^2 + \mathbf{a}_6 (\mathbf{k}_x)^2 + \mathbf{a}_7 k_y k_z + \mathbf{a}_8 k_y (-\mathbf{k}_x) + \mathbf{a}_9 (-\mathbf{k}_x) k_z + \dots$$

	Before	After
k_y	a_2	a_1
k_x	a_1	$-a_2$
k_z	a_3	a_3
k_y^2	a_5	a_4
k_x^2	a_4	a_5
k_z^2	a_6	a_6
$k_y k_z$	a_9	a_7

$$\begin{array}{lll} k_y k_x & a_8 & -a_8 \\ k_x k_z & a_7 & -a_9 \end{array}$$

From this we can say that,

$$a_4 = a_5 = a \text{ (say)}$$

$$a_6 = a_6 = b \text{ (say)}$$

$$a_1 = a_2 = a_3 = a_7 = a_8 = a_9 = 0$$

Hence we write,

$$\mathbf{E} = \mathbf{a}_0 + \mathbf{a}_3 \mathbf{k}_x + \mathbf{b}(\mathbf{k}_x^2 + \mathbf{k}_y^2) + \mathbf{a}_6 \mathbf{k}_z^2$$

Then sending x into z gives us $a_6 = a_5 = a_4$, hence we get,

$$\mathbf{E} = \mathbf{a}_0 + \mathbf{b}(\mathbf{k}_x^2 + \mathbf{k}_y^2 + \mathbf{k}_z^2)$$

This looks like a free electron! This is called the "nearly free electron model".

The majority of solids that occur in nature have symmetry operations that send x into y and x into z. This means to the mathematics that there are no linear terms left in the expression which cause the expression to behave like the free electron model.

We choose the b parameter to resemble $\hbar^2/2m$, we actually get,

$$\mathbf{b} = \hbar^2/2m$$

Where m^* is the effective mass. The electron's mass does not really change, but it is used so that the b parameter looks like a known physical quantity. This confusion is the main flaw of the experimentalist's solution !

If for a crystal x into y and x into z are both symmetry operations, we get the energy as,

$$\mathbf{E} = \mathbf{a}_0 + \mathbf{b}(\mathbf{k}_x^2 + \mathbf{k}_y^2 + \mathbf{k}_z^2)$$

This is called a **direct band gap semiconductor**

If x into y and x into z are not both symmetry operations we get the energy as,

$$\mathbf{E} = \mathbf{a}_0 + \mathbf{a}_1 \mathbf{k}_x + \mathbf{a}_2 \mathbf{k}_y + \mathbf{a}_3 \mathbf{k}_z + \mathbf{a}_4 \mathbf{k}_x^2 + \mathbf{a}_5 \mathbf{k}_y^2 + \mathbf{a}_6 \mathbf{k}_z^2 + \mathbf{a}_7 \mathbf{k}_x \mathbf{k}_y + \mathbf{a}_8 \mathbf{k}_y \mathbf{k}_z + \mathbf{a}_9 \mathbf{k}_x \mathbf{k}_z$$

This is called an **indirect band gap semiconductor**

The confusion about effective mass also comes in when they rewrite the coefficients as,

$$\begin{aligned} \hbar^2/2m_x &= a_4 k_x^2 \\ \hbar^2/2m_y &= a_5 k_y^2 \\ \hbar^2/2m_z &= a_6 k_z^2 \\ \hbar^2/2m_{xy} &= a_7 k_x k_y \\ \hbar^2/2m_{yz} &= a_8 k_y k_z \\ \hbar^2/2m_{xz} &= a_9 k_x k_z \end{aligned}$$

These are called the "effective mass tensors". For an electron,

$$1/m_x = 1/m_y = 1/m_z$$

Experimentally it is found that for many solids (not all) this expansion can be truncated at the second order to a good approximation. There are some solids however where this is not a good approximation.

23/02/1999

Direct and Indirect bandgaps in bulk (ie 3D) materials

For simplicity we shall neglect all three dimensions in mathematical analysis. In general here,

$$E = A + Bk_x^2$$

We look for point group symmetry, ie there is no linear terms (no terms in k_x).

When we form Zinc sulphide (for example), bringing together zinc and sulphur atoms. When atoms are brought together in a solid the individual atomic energy levels are broadened out into bands and generally the bands do **not** overlap in energy value.

In this example, zinc has filled inner shells and two electrons in an outer shell. When the energy levels are broadened, the filled ones must still be filled. Sulphur has filled inner shells and its outer shell is short of two electrons from being full. The outer shell (band) of the sulphur atom is lower than the outer shell of the zinc atom, so the two electrons on the outer zinc shell go to fill up the outer shell of sulphur atom, they go to a lower energy state. The recently vacated outer zinc shell is now the *next empty state* of the compound. In general we do not consider the filled inner shells (apart from X-ray spectroscopy). All the physics in semiconductor physics is concerned with the highest filled electron energy level and the next unfilled energy level. The highest filled level is termed the **valence band** and the next unoccupied level is called the **conduction band**. The energy difference between the valence and the conduction band is called the **band gap**. In atomic terms an electron traversing the band gap actually means that it has travelled from one atom in the compound to a different

atom. For our example an electron moves from a sulphur atom to a zinc atom when it is excited to the conduction band. An electron in the conduction band can move from one zinc atom to the next without any (internal) potential energy penalty, this forms an electrical current in the compound.

Electrons can move in the valence band by electrons from one sulphur atom filling the gap in another sulphur atom. Experimentalists use a model to describe this as holes moving around, entities that have a positive charge and annihilate electrons, negative charge carriers, on contact. When electrons move in the valence band, many move at once, it is difficult to consider this many electrons moving at once, if we consider the movement of gaps in the electron population, it is easier to conceptualise. The movement of electrons and gaps (holes) are different and they even move in opposite directions, it is said they have *different mobilities*. The symbol \mathbf{n} is used to denote the electron density and the symbol \mathbf{p} is used to denote the hole concentration. Both electrons and holes contribute to the electric current (remember, both are simply electron movements!).

Going back to the valence/conduction band model. If we have an electron high in the conduction band it will *sink* to the bottom of the conduction band. If we then were to excite an electron to the conduction band from the valence band then we create a hole in the valence band. All of the electrons in the valence band will fall and occupy lower empty states, this creates the effect of a hole rising up the valence band.

Recalling our energy expression

$$\mathbf{E}=\mathbf{A}+\mathbf{Bk}^2$$

This is of the same form in either band, but the coefficient is different in each band because each band is across different atoms. If we were to plot the electron energy of both atoms we would have two parabolas. In k-space, at $k=0$, we have the direct energy gap between the parabolas

fig 1

However in the valence band, we now consider holes, so the energy is of opposite sign so the lower parabola is *turned upside down!*

fig 2

So it can be clearly seen that band gap is the separation of the minima of the energy parabolas in k-space.

Indirect band gap

Here if we have linear terms due to asymmetry, we write,

$$E=A+Bk+Ck^2$$

As k goes to zero E goes to A and k goes to \pm infinity so does E . To find the value of A we find the turning point of the k parabola.

$$dE/dk=B+2ck=0$$

$$\text{Therefore,}$$

$$k=-B/2C$$

If we consider shifting the parabola along the k axis, ie we change coordinate systems.

$$k=k'-a$$

$$E=A+Bk+Ck^2$$

$$E=A+B(k'-a)+C(k'-a)^2$$

$$=A+Bk-Ba+C(k'^2+2ak+a^2)$$

$$=A+Bk-Ba+Ck^2+2Cak+Ca^2$$

Now let,

$$B=2Ca$$

$$a=B/2C$$

Doing this now gives us,

$$E=A'+Ck'^2=A'+Ck^2$$

Where,

$$A'=A+a^2C-Ba$$

Now, the electron's energy in both bands are now displaced from each other, the minimums do not lie along the same k value. The band gap is now said to be **indirect**. It is indirect because the band gap now lies along a diagonal line in the E - k space. The use of this leads us to the next section.

Photoluminescence

Many devices in modern technology are concerned with emitting or absorbing photons.

Consider a photon hitting, or **colliding**, with a zinc atom. An outer electron is excited and moves off to another zinc atom, it moves through the conduction band, and another photon is emitted. This is a collision, therefore energy and momentum are always conserved.

fig 3

Absorption is simply a special case of a collision. In the case of absorption, the frequency of the emitted photon happens to be zero. This is also simply the reverse of emission, the two are the same process. All processes involving photons are scattering processes in which momentum is conserved.

Recall the De Broglie relation,

$$\mathbf{p} = \frac{h}{\lambda} \mathbf{a} \\ = \frac{h}{\lambda} (\mathbf{v}/v)$$

Also from Planck,

$$\mathbf{E} = h\nu$$

Hence the two can be combined,

$$\mathbf{p} = \mathbf{E}/c$$

Because of the high magnitude of c , p is approximately zero, which implies virtually zero mass for a photon.

Consider the E-k diagram for a direct band gap in figure 4. If $p = \hbar k$ then for pure photon absorption p is zero so for the transition from the valence band to the conduction band, there is no change in the momentum (because none is gained from photon) so the k vector remains constant and the transition is vertical in the diagram.

fig 4

For indirect band gap semiconductors emission and absorption is weak. This is because there is a difference in the k vector (and hence momentum) between the minimum energies and photons cannot provide enough momentum to give the transition.

fig 5

This means that indirect band gap semiconductors are very poor for photoluminescence applications. Direct band gap semiconductors are far better.

Einstein's theory of stimulated emission and absorption

If an electron is desired to be sent from a lower energy to a higher energy. If there is a certain probability of an electron absorbing a photon to go to the higher energy. The more photons that are sent to *simulate* the electron the higher the chance of an absorption event. After the excitation event there are $N-1$ photons. If we let the process run in reverse we would see the electron come down and be stimulated by the $N-1$ photons to emit another photon. This is true

even if $N=1$ we would see the electron being stimulated by the vacuum. Einstein called this **stimulated emission**. This is fundamental concepts of lasers.

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Scattering rates and Final density of states
(Transition rate)

Suppose we have some initial state, i , and we goto some final state, j , by means of some scattering process. We want to know how quickly we can get from state i to state j , i.e. the scattering rate. A general example would be how quickly an electron emits a photon after a collision. Quantum mechanics tells us that the scattering is proportional to the density of final states at j . The density of final states is conventionally written as ρ .

We now need to take some techniques from Statistical mechanics. Suppose we have a box with dimensions dx, dy and dz . It has some arbitrary position in space. We want to find the probability that in x to $x+dx$ and y to $y+dy$ and z to $z+dz$ you would find a 'particle' whose momentum lies between p_x to p_x+dp_x , p_y to p_y+dp_y and p_z to p_z+dp_z . The answer will be some number dN .

$$dN=(dx \cdot dp_x/h) \cdot (dy \cdot dp_y/h) \cdot (dz \cdot dp_z/h)$$

Where h is Plank's constant.

Heisenburg says that if we confine a particle to a region dx , the following is true,

$$dx \cdot dp_x \Rightarrow h$$

So the minimum value of the product $dx \cdot dp_x$ is h . The " $dx \cdot dp_x$ ", " $dy \cdot dp_y$ ", " $dz \cdot dp_z$ " is called a **phase space**. In terms of the quantities included it is called a six dimensional 3-D phase space (for bulk material). So rewriting dN (for bulk material) we have,

$$dN=(dx \cdot dy \cdot dz) \cdot (dp_x \cdot dp_y \cdot dp_z)/h^3$$

$$dN=V \cdot dp_x \cdot dp_y \cdot dp_z/h^3$$

Where V is the total volume of the system and dN' is the number per unit volume

$$dN'=dN/V=dp_x \cdot dp_y \cdot dp_z/h^3$$

If we were to integrate over space we could use cartesian coordinates, dx, dy, dz or polar coordinates,

$$dV=dx dy dz$$

$$dV=4\pi r^2 dr$$

In the second, polar coordinates, $4\pi r^2$ is the surface of a sphere and dr is the thickness of a spherical shell. We can therefore write,

$$dN' =$$

In a solid we can write,

$$E = p^2 / 2m$$

(Where m is the effective mass).

We can go onto to then say,

$$\begin{aligned} dE &= pdp / m \\ pdp &= m dE \end{aligned}$$

but,

$$p = (2m)^{1/2} E^{1/2}$$

$$\begin{aligned} dN' &= (4\pi / h^3) \cdot (2m)^{3/2} \cdot E^{1/2} \cdot m \cdot dE \\ dN' &= (4\pi \cdot 2 \cdot \pi \cdot (m)^{3/2} \cdot E^{1/2} \cdot dE / h^3 \end{aligned}$$

dN' / dE = density of states per unit volume per unit energy interval.

$$p(E) = dN' / dE = 4 \cdot 2\pi (m)^{3/2} \cdot E^{1/2} / h^3$$

$$p(E) \propto E^{1/2}$$

If we set off with N atoms per unit volume and bring them together from infinity we have N energy levels before we and we should have N energy levels after, but they are now broadened.

$$\text{Integ } (E_1, E_2) \{p(E) dE\} = N$$

The density of states at the bottom of the band is low (*this is bad news for photoluminescence, eg lasers*), but is very large for large values of E , so the density of states at the top of the energy band is very high (*this is bad news for non-radiative decay processes, ie low quantum efficiency*).

Suppose we have a laser where we excite an electron into the conduction band, creating a electron-hole pair which then recombines with the hole in the valence band thus creating a photon. The density of states in the lower levels is low though, so the radiative transition rate is low. This means that we have fewer photons per unit time. To restate, this is because the density of final states is low.

Now look at another system. Let us suppose that we have a semiconductor which has a high energy electron which impacts the valence band. This collision hits a valence electron which is then

excited into the conduction band (there is no photon created), the impacting electron rebounds away with a lower energy. If this process is ran backwards, this is known as a **Auger process** In 3D Auger processes goto final electrons with high energy which means large density of states which means it is highly likely to happen. These processes are bad news. When the high energy electrons are released, the radiative rate goes down and energy is the dissipated in the lattice.

If we now go to two dimensional structures, we have,

$$\begin{aligned} dN &= (dx dp_x / h) (dy dp_y / h) \\ dN &= A \cdot dp_x dp_y / h^2 \end{aligned}$$

We also now consider a circle instead of a sphere for polar coordingates.

$$dN' = dp_x dp_y / h^2 = dN / A$$

$$A = 2\pi r dr$$

$$dN' = (2\pi i / h^2) m dE$$

$$p = dN' / dE = 2\pi i m / h^2 = \text{constant}$$

$$\text{Integ } (E_2, E_1) \{p(E) dE\} = N$$

Since the density of states is constant, there are more states at high levels and fewer at higher energies. So the radiative rate goes up by large factors and the nonradiative (Auger) goes down by large factors. All this is gained by having a two dimensional structure rather than three dimensional. Now let us take this further, to one dimension.

In 1D,

$$\begin{aligned} dN' &= dx dp_x / h \\ &= L dp_x / h \\ &= dN / L = dp_x / h \end{aligned}$$

$$\begin{aligned} E &= p^2 / 2m > p_x = (2m)^{1/2} E^{1/2} \\ dE &= p_x dp_x / m \end{aligned}$$

$$\begin{aligned} dp_x &= m dE / p_x \\ &= (m^{1/2} / 2^{1/2} m) E^{-1/2} \cdot dE \end{aligned}$$

$$dN' / dE \propto E^{1/2}$$

So we can see the pattern,
Bulk Well Wire Dot

$$dN'/dE \propto \begin{matrix} 3D & 2D & 1D & 0D \\ E^{1/2} & E^0 & E^{-1/2} & E^{-1} \end{matrix}$$

In Gallium Arsenide semiconductors, someone called Gunn found something unusual. In general when the voltage is increased the current increases. Gunn found that this was true upto a critical voltage, but beyond this he found that the current pulsed at high voltages. In most semiconductors the charge carriers have low energy. When carriers are sat near the bottom of the conduction band (or the 'holes' are at the top of the valence band). These carriers are said to be *COOL*. Gunn was dealing with *HOT* carriers.

Most experiments dealt with low k (low momentum) carriers, in general recall $E=Ak^2$. However the energy could be of the form,

$$E=ak^2+bk^3+ck^4$$

But for small values k^3 and k^4 are tiny. As;

$$\begin{matrix} k \rightarrow 0 & E \rightarrow 0 \\ k \rightarrow \infty & E \rightarrow k^4 \rightarrow \infty \end{matrix}$$

To determine when these occur we find the turning points,

$$\begin{aligned} dE/dk &= 2ak - 3bk^2 + 4ck^3 \\ &= k(2a - 3bk + 4ck^2) \\ &= 0 \end{aligned}$$

fig 1

Cool carriers only really exist around the minimum around $k=0$. Gunn pushed the carriers to explore the rest of the polynomial structure. Their momentum increased but their energy decreased. We now wish to explore the consequences of this.

From classical mechanics,

$$\begin{aligned} v &= p/m = \hbar k / m \\ E &= \hbar^2 k^2 / 2m = Ak^2 - bk^3 + ck^4 \\ 1/m &= (2/\hbar^2 k^2)(ak^2 - bk^3 + ck^4) \\ &= (2/\hbar^2)(a - bk + ck^2) \end{aligned}$$

This is called **non-parabolicity** is not a constant, it varies with k and hence varies with E . So writing our velocity expression again,

$$\begin{aligned} v &= \hbar k / m \\ v &= (\hbar k / m) \cdot (k/k) \cdot (2/2) \\ v &= (\hbar^2 k^2 / 2m) \cdot (2/k) = (2/k) \cdot E \\ v &= (2/k)(ak^2 - bk^3 + ck^4) \end{aligned}$$

$$v=2(ak-bk^2+ck^3)$$

But if $k \ll 1$, $v \propto k$
 $k \rightarrow 0$ $v \rightarrow 0$
 $k \rightarrow \infty$ $v \rightarrow k^3 \rightarrow \infty$

$$\frac{dv}{dk}=2(a-2bk+2ck^2)=0$$

$$(k=k_1, k=k_2)$$

fig 2

From Newton's law, increasing force increases momentum. Also current is number of carriers times the average current. If the velocity decreases then so does the current. At high voltages the momentum increases and the effective mass decreases so much that the velocity decreases which in turn reduces the current, even as the voltage increases. Until the Gunn effect was discovered all conductances were positive, given dI/dV it was said that differential conductances were always positive. But in such semiconductors the differential conductance was negative.

Consider the equation of continuity from quantum mechanics,

$$(\frac{dp}{dt})+(\frac{dj}{dx})=0$$

and Poisson's equation,

$$\partial^2\phi/\partial x^2=-\rho/E$$

and Ohm's law,

$$j=-\partial \cdot d\phi/dx$$

Putting these together gets us,

$$dp/dt=-dj/dx$$

If ∂ is a constant,

$$\frac{dp}{dt}=-\frac{d}{dx}(-\partial \cdot \frac{d\phi}{dx})$$

$$=\partial \cdot \partial^2\phi/\partial x^2$$

$$dp/dt=-\partial \rho/E$$

$$\rho=\rho_0 \cdot \exp(-\partial t/E)$$

If ∂ is negative the charge fluctuation grows exponentially quickly with the time. This is what happens in the Gunn effect. Very large internal dipoles

fig 3

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Donors - Acceptors and Impurities

Recall that in semiconductors we have a filled *valence band* and an empty *conduction band*. In semiconductor devices we are concerned with electrical properties. Pure semiconductors has poor electrical conductivity. An electron can only participate in a physical process if there are unoccupied energy levels available to it nearby. For example, Sodium is a metal with filled inner shells and one electron in it's conduction level (which we shall say has a height of 1eV), whereas two would fill it. So the outer level is only half full. At room temperature the energy is of order kT which is about 1/40 eV. So the only electrons that could ever move are the ones that within 1/40 eV of the next unoccupied energy state. Statistically that is only about 1 to 2 percent. This for an anomaly observed in the classical era. When measuring the conductivity of Sodium, since it had a single valence electron they expected to see a current of magnitude expected when there are as many electrons as atoms, the observed current was much lower. That was because there was of about 1/40 of the electrons could actually move.

Impurities

If we have a material with n outer electrons. If we add an impurity that increases this number by one it is called a **donor** If we add an impurity that decreases n it is called an **acceptor**

When a donor is added we imagine simply that we have a lattice of an original material (like silicon) and replace one atom with a donor atom. This donor atom has an extra electron and positive charge compared to the rest of everything. Say the extra electron would normally (meaning if it was free and not seeing the donor) go into the conduction band, but it can then see the extra positive charge on the donor atom, the charges interact and the energy of the donor electron is lowered, this creates a lower energy band which sits just below the conduction band. The opposite is true for acceptors, they create an energy level that is just above the valence band. The energy difference between the donor level and conduction band and acceptor level and valence band is the same. It is called the binding energy is denoted by **ϵ_a** , and,

$$kT \gg \epsilon_a$$

Chemical potential, μ

The chemical potential, μ , is a measure of the energy required to create a 'particle' in the system. At zero Kelvin the chemical potential lies half way between the uppermost occupied energy level of

any significance and the next unoccupied energy level of any significance. i.e. This energy is half way between the conduction and valence level, it is known to experimentalists is known as the fermi level.

The chemical potential always lies half between the levels, but for metals this energy lies at almost the same level at the uppermost occupied levels is because the energy levels are so close that the chemical potential looks like it is at the top of the uppermost occupied level.

What happens to μ if donors are added to the system ? A donors are added the chemical potential rises because it is harder to create/add electrons when there are donor due to the electrons orbiting the donor. Likewise if acceptors are added the chemical potential falls because there are more 'holes' which make it easier for electrons to be created/added.

Increasing the dopant concentration until the orbits start to overlap leads to a state where we say the semiconductor is **degenerately doped**. This means that there are so many energy levels present we have a continuum or a band. When we have a high concentration of donors (for example) we start to get an energy band below the conduction band, adding more donors we get an overlapping of donor and conduction band. Since the chemical potential lies between occupied and unoccupied levels, it will begin to lie above the donor level. In a degenerately doped n-type semiconductor the chemical potential lies in the conduction band.

Esaki Tunnel Diode

Let us first revise our p-n junction knowledge.

Consider a p type semiconductor and an n type semiconductor, of which neither are degenerately doped.

fig 1

When the two are joined together, there will be a transfer of particles until the chemical potential is the same.

fig 2

That process gives rise to a **contact potential**. This is due to the transfer of particles. As electrons are forced to move across, it gets harder for each electron to move across because they are being repulsed by the electrons that went across before them. We get a **depletion zone** due to this particle transfer. The depletion zone width decreases as the density of either acceptor

or donor concentration increases. If we apply a *reverse bias* to the junction, we force electrons from right to left the energy difference between regions increases, this can continue until it is impossible for electrons to climb the potential hill. This is covered well in most text books.

Back to Esaki!

Esaki doped a pn degenerately, so much that the depletion region was so small that there was an appreciable tunneling current

fig 3

Because of the degenerate doping there are holes in the valence band and electrons in the conduction band.

fig 4

The tunnel current determined the behaviour in fig 3. We actually have current decreasing with increasing voltage. This meant we have differential resistance which lead to microwave oscillators. This works for an semiconductor you can both n-type and p-type degenerately dope.

16/03/1999

Quantum Wells

So far we have discussed 3 dimensional structures, ie three large dimensions ($\sim 1\mu\text{m}$), now we discuss what happens when we reduce one of the dimensions (to around 100\AA).

Consider Gallium Arsenide, then we replace some of the Gallium with some aluminium

GaAs \rightarrow GaAlAs

It is found that the band gap is then increased with the aluminium substitution. When this material is made in practice, it is grown in sandwiches, below is how it would look in real space.

GaAs | GaAlAs | GaAs | GaAlAs | GaAs | GaAlAs

The GaAs region is called the narrow(er) band gap and the GaAlAs is called the large(r) band gap. Each region is only 100\AA thick, ie it is very small and very difficult to produce. Let us consider how these fit together on an energy basis. The smaller band gap can either fit inside the larger one or not. If it does it called a **type 1** quantum well and if it doesn't it is called a **type 2** quantum well.

fig 1**fig 2**

Figure 2 shows the energy profile for an electron traveling through a type 1 semiconductor/quantum wells. Figure two also shows us that in a type 1 structure the electrons and the holes sit in the same semiconductor, in the same region of space.

fig 3

Figure 3 shows the energy profile for an electron traveling through a type 2 semiconductor/quantum wells. Figure three shows us that in a type 2 semiconductor the electrons and the holes are spatially separated (in real space that is).

By drawing an analogy to bulk semiconductors, in energy - k space we have direct and indirect band gaps. In the former electrons and holes can recombine (*radiative decay*) very easily, and in the latter it is difficult, they need phonon assistance to perform radiative decay. So we can see that type 2 semiconductors are poor radiative emitters and type 1 semiconductors are good radiative emitters.

Multiquantum wells and Super lattices

For this topic we will consider a type 1 semiconductor. While looking at the electrons in the conduction band of the type 1, we can see that we have a series of quantum wells, wells because of the energy profile and *quantum* because of the small size, the spaces between are called *barriers*. The wells are always of quantum order of size, but the barriers are not necessarily of quantum size.

If we perform a thought experiment on the barriers size now. Let us expand the barrier to a great distance. Quantum mechanics tells us that the wave function of a particle can tunnel into the barrier (while exponentially decaying). If the barrier is of such a size that the wave function of particles in each well cannot penetrate the next well, we have ***multiquantum wells*** because the wave functions in each well are ***isolated***. If however now, the barrier is reduced (to quantum dimensions) so that wave functions from each begin to overlap, particles in each well interact we have what is called a ***super-lattice***, the band gap in a super-lattice is termed a ***mini-band***.

An n well multiquantum well is quantum wells with a great separation and an n well super lattice is wells with a small separation. You can control the properties of the miniband in a semiconductor by varying the well and the barrier width.

We wish to have a quantitative understanding we now employ quantum mechanics. Say we have a type 1 well of depth approximately 300

mev, we will assume the depth is infinite, this will simplify our analysis but still demonstrate the essential physics. The well also has a width d . For the infinite barrier model, **$\Psi=0$ at the edges of the well.** For solutions here we must have harmonic/standing waves that fit within the width of the well. So the condition for standing waves is,

$$d=n(\lambda/2)$$

We have our zero energy at the bottom of the well and we also have no potential energy, so we just have kinetic energy relative to the bottom of the well. So,

$$E=KE+PE$$

$$E=KE$$

$$E=p^2/2m$$

$$p=\lambda/h \quad \text{De Broglie}$$

$$\text{Energy}=\frac{h^2}{2m\lambda^2}$$

$$1/\lambda = n/2d$$

$$E=(\frac{h^2}{2m})\cdot(\frac{n^2}{4d^2})$$

$$E=(\frac{h^2}{8md^2})n^2$$

$$E=n\lambda\cdot n^2$$

$$n\lambda=\frac{h^2}{8md^2}$$

Envelope function Approximation

This is based on the following argument. Suppose we have a bulk material, atoms separated by say one angstrom. For an electron travelling along such a lattice, there are only approximate solutions, based on a plane wave,

$$A\cdot\exp(ikx)$$

It is supposed that there are no atoms present (this is not a real construct), we write,

$$\Psi_{\text{solid}}=A\exp(ikx)\cdot f(x)$$

but,

$$f(x\rightarrow\infty)=1$$

and since we are in a periodic lattice we can say,

$$f(x+a)=f(x)$$

This is **Bloch's theory**. Now consider the given hamiltonian,

$$\text{hamiltonian} = \mathbf{p}^2/2\mathbf{m} + \mathbf{V}(\mathbf{x})$$

(where m is the free electron mass). So we can then write,

$$\begin{aligned} (\mathbf{p}^2/2\mathbf{m} + \mathbf{V}(\mathbf{x}))\Psi &= \mathbf{E} \Psi \\ (\mathbf{p}^2/2\mathbf{m} + \mathbf{V}(\mathbf{x})) \exp(\mathbf{k}\mathbf{x}) f(\mathbf{x}) &= \mathbf{E} \exp(\mathbf{k}\mathbf{x}) f(\mathbf{x}) \end{aligned}$$

through rigorous mathematical manipulation we can eliminate,

$$(\mathbf{p}^2/2\mathbf{m} + \mathbf{V}(\mathbf{x})) \exp(\mathbf{k}\mathbf{x}) = \mathbf{E} \exp(\mathbf{k}\mathbf{x})$$

Where m^* is the effective electron mass. the $\exp(\mathbf{k}\mathbf{x})$ is a slowly varying function because k is small.

We introduce a rapidly varying function, $\mathbf{u}(\mathbf{x})$, which varies rapidly describing the potential along each atom, we write,

$$\exp(\mathbf{i}\mathbf{k}\mathbf{x}) * \mathbf{u}(\mathbf{x})$$

Plotting this would give the periodic function (u) damped by the slowly varying function (exp), as an envelope, in harmonic terms.

The envelope approximation assumes that an electron in a quantum well has slowly varying function which can be solved for by using the effective mass approximation.

In 3D we solve the equation,

$$\begin{aligned} ((-\hbar^2/2\mathbf{m}) * \partial^2/\partial \mathbf{x}^2 + \mathbf{u}(\mathbf{x}))\Psi &= \mathbf{E} \Psi \\ -((\hbar^2/2\mathbf{m})\partial^2/\partial \mathbf{x}^2 + \mathbf{V}(\mathbf{x}))\Phi &= \mathbf{E} \Phi \end{aligned}$$

In 2D we have,

$$\begin{aligned} ((-\hbar^2/2\mathbf{m}) * \partial^2/\partial \mathbf{x}^2 + \mathbf{u}(\mathbf{x}))\Psi &= \mathbf{E} \Psi \\ ((-\hbar^2/2\mathbf{m}) * \partial^2/\partial \mathbf{x}^2 + \mathbf{V}(\mathbf{x}))\Phi &= \mathbf{E} \Phi \end{aligned}$$

Where Φ is the envelope function. Now for the infinite well approximation,

$$\begin{aligned} ((-\hbar^2/2\mathbf{m}) * \partial^2/\partial \mathbf{x}^2 + \mathbf{V}=0)\Psi &= \mathbf{E} \Psi \\ ((-\hbar^2/2\mathbf{m}) * \partial^2/\partial \mathbf{x}^2 + \mathbf{V})\Psi &= \mathbf{E} \Psi \\ ((-\hbar^2/2\mathbf{m}) * \partial^2/\partial \mathbf{x}^2)\Psi &= (\mathbf{E} - \mathbf{V})\Psi = \mathbf{E}' \Psi \end{aligned}$$

$$\begin{aligned} \partial^2\Psi/\partial \mathbf{x}^2 &= \alpha^2\Psi \\ \alpha^2 &= 2\mathbf{m}\mathbf{E}' / -\hbar^2 \end{aligned}$$

Let $\Psi = y$

$$\partial^2 y / \partial \mathbf{x}^2 = \alpha^2 y$$

The solution is,

$$y = A \sin(ax) + B \cos(ax)$$

$y=0$ when $x=0$, ie $B=0$.

Hence,

$$y = A \sin(ax)$$

$y=0$ when $x=d$,

$$0 = A \sin(ad) \rightarrow ad = n\pi \text{ (n is integer)}$$

1)

$$\begin{aligned} (2mE'/\hbar^2) &= n^2\pi^2 \\ E' &= (n^2 m^2 \hbar^2 / 2\pi^2 d^2) \\ &= (n^2 \pi^2 / 2m d^2) (\hbar^2 / 4\pi^2) \\ &= (\hbar^2 / 8m d^2) n^2 \end{aligned}$$

$\Psi = A \sin(ax)$ where a varies with E .

The total wave function is $\Psi = u(x) \cdot \sin(ax)$, where $u(x)$ varies rapidly.

For electron recombining, where \mathbf{A} is a vector potential,

$$\begin{aligned} \mathbf{p} &\rightarrow \mathbf{p} - (e/c)\mathbf{A} \\ p^2/2m &\rightarrow p^2/2m - (e/mc)\mathbf{A} \cdot \mathbf{p} - (e/mc)\mathbf{p} \cdot \mathbf{A} \\ p^2/2m &\rightarrow p^2/2m - e/mc \mathbf{A} \cdot \mathbf{p} \\ &\rightarrow \text{kinetic} + \text{operator} \end{aligned}$$

operator $\partial/\partial x$

Transition rate (matrix element)

$$\Psi_{i_{\text{initial}}} = \Psi_{i_{\text{final}}}$$

Matrix element = $\text{Integ} () \{ \mathbf{P} \cdot \partial/\partial \mathbf{x} \} \Psi_i$

(if any of this was required in an exam, it would be given!!!).

Say we have an electron recombining with a hole, Ψ_i describes it in the conduction band and Ψ_f describes it in the valence band.

$$\begin{aligned} \Psi_i &= u_e(\mathbf{x}) \cdot \phi \\ \Psi_f &= u_h(\mathbf{x}) \cdot \phi \end{aligned}$$

u varies rapidly and ϕ varies slowly. Now we define the matrix element,

$$\begin{aligned} \text{matrix element} &= \text{Integ} () \{ (u_f \cdot \phi) \partial/\partial \mathbf{x} (u_e \cdot \phi) d\mathbf{x} \} \\ &= \text{Integ} () \{ (u_f \cdot \phi) (\partial/\partial \mathbf{x} (u_e \cdot \phi) + u_e \cdot \partial/\partial \mathbf{x} (\phi)) d\mathbf{x} \} \\ &= \text{Integ} () \{ (u_f \cdot \partial u_e/\partial \mathbf{x} \cdot \phi(x) + u_e(x) \cdot u_f(x) \cdot \phi(x) \cdot \partial \phi/\partial \mathbf{x}) d\mathbf{x} \} \end{aligned}$$

$$= \left(\int \{u/\partial x \cdot dx\} * \int (\phi \cdot h dx) \right) + \left(\int \{u(x)u(x)dx\} * \int \{Q(x)(\partial Q/\partial x)dx\} \right)$$

$$V(x) = P * \int \{Q \cdot dx\}$$

This is the expression for the internad transition, which can be verified by experiment. In the $\int \{u \cdot (\partial Q/\partial x)\}$ term is zero and u_e is a function of a free electron at $u=0$. The various energies are orthogonal to one another and therefore is zero.

We also consider the inter

Recap

Envelope functions approximation

Bulk - Bloch's theorem,

$$\Psi(\mathbf{r}) = \exp(i\mathbf{k} \cdot \mathbf{r}) * u(\mathbf{r})$$

$$u(\mathbf{r} + \mathbf{a}) = u(\mathbf{r})$$

$$\mathbf{a} = \text{lattice vector}$$

We wish to eliminate u_k , we use the Schrödinger equation,

$$\left[\left(-\frac{\hbar^2}{2m} \nabla^2 + U(\mathbf{r}) \right) \Psi = E \Psi \right]$$

$$\left[\left(-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}) \right) \phi = E \phi \right]$$

$$\phi = \exp(i\mathbf{k} \cdot \mathbf{r}) - \text{Bulk}$$

$V(\mathbf{r}) = \text{slowly varying potential at the bottom of a given band}$
 $= \text{const.}$

Doing this is known as effective mass theory

When we bring different semiconductors together, we get a periodic potential, the difference in the potentials being V_B ,

$$\left(\left(-\frac{\hbar^2}{2m} \nabla^2 + V=0 \right) \phi = E \phi \right)$$

$$\left(\left(-\frac{\hbar^2}{2m} \nabla^2 + V_B \right) \phi_B = E \phi_B \right)$$

We then have,

$$\Psi = \phi U$$

Where ϕ is the envelope function.

13/04/1999

We are interested in the optical properties of the semiconductor

combinations. We wish to determine the "allowed" transitions, ie the selection rules.

fig 1

We use something called the Transition Operator which looks something like this,

A.p

Where A is the vector potential of the E.M. field (ie photons). p is the linear momentum operator,

$$\text{Transition rate} \propto \left| \int \Psi_i \cdot \mathbf{A} \cdot \mathbf{p} \cdot \Psi_f \cdot dT \right|^2$$

$$\propto \left| \int \Psi_i \frac{\partial}{\partial z} \Psi_f \cdot dT \right|^2$$

$$\Psi_i = \phi_i u_i$$

$$\Psi_f = \phi_f u_f$$

$$\frac{\partial}{\partial z} \Psi_i = \left(\frac{\partial \phi_i}{\partial z} \right) u_i + \phi_i \left(\frac{\partial u_i}{\partial z} \right)$$

$$\int \Psi_i \frac{\partial}{\partial z} \Psi_f = \int \left(\frac{\partial \phi_i}{\partial z} \right) \left(\frac{\partial u_f}{\partial z} \right) u_f dz$$

$$= \int \left(\frac{\partial \phi_i}{\partial z} \right) \phi_f \left(\frac{\partial u_f}{\partial z} \right) dz$$

$$\int f(z)g(z)dz = \int f(z) \cdot \int g(z)dz$$

If g(z) is rapidly varying f(z) is slowly varying

$$= \int \left(\frac{\partial \phi_i}{\partial z} \right) dz * \int u_f \cdot dz + \int \phi_i dz * \int u_f \left(\frac{\partial u_f}{\partial z} \right) dz$$

Infra Red detectors or Infra red lasers.

Because the structure of such materials is periodic initial and final states of functions become equal (for inter sub band transitions), so we have the general quantum mechanics results that certain integral factors are equal to 1. So we come to the conclusion,

$$\text{Transition Rate (TR)} \propto \int \left(\frac{\partial \phi}{\partial z} \right) \phi$$

Also recall,

$$\phi_{\text{even}}(-z) = \phi_{\text{even}}(z)$$

$$\phi_{\text{odd}}(-z) = -\phi_{\text{odd}}(z)$$

fig 2

From the argument in figure two, the rule of thumb is that the allowed transitions are even to odd (odd and even energy levels that is).

Interband Transitions

$\int u_f u_i dz = 0$ because u_f and u_i are eigen functions belonging to

different energy levels

$$\text{Integ } (\mathbf{u}(\partial/\partial z)\mathbf{u}) = \text{some number.}$$

$$\text{Integ } (\mathbf{00},dz)$$

Here odd to even transitions become zero. Even to Even transitions give some value and hence exist. Odd to Odd transitions give a value also. This means that for interband transitions only odd to even transitions are phorbidden.

Quantum Well lasers - Solid State

Photons stimulate absorption and emission (Einstien). If we send in photons that exactly match the energy gap between levels. The stimulated absorption is proportional to the number of photons, N.

fig 3
fig 4

Einstein thought that emission was simply the reverse of absorption. However going through the N and N-1 photon levels, when N=1, we have **spontaneousemission**

fig 5

Radiation - $\rho(\nu)$ this is energy density per unit frequency. Einstien suppose he had a two level atom.

fig 6

The stimulated absorption rate is $W_{12}=B_{12}\rho(\nu)$
The stimulated emission rate is $W_{21}=B_{21}\rho(\nu)$
The spontaneous emission (down) rate is A_{21}

$$\text{down rate} = N(B_{21}\rho(\nu)+A_{21})$$

$$\text{up rate} = NB_{12}\rho(\nu)$$

At equilibrium up rate=down rate,

$$N_2(B_{21}\rho(\nu)+A_{21}) = N_1B_{12}\rho(\nu)$$

Einstein guessed B_{21} , A_{21} and B_{12} are properties of the system, they do not depend on the radiation. If they do not depend on the radiation field choose a particular radiation field, in particular, black body radiation. This is because Plank had just determined the $\rho(\nu)$.

$$\rho(\nu)=(8\pi h\nu^2/c^3)(1/(\exp(h/kT)-1))$$

This expression is for a system in thermal equilibrium. In which case Boltzmann statistics applies, which state the density of energy states

is,

$$p(E) \propto \exp(-\beta E)$$

$$\beta = 1/kT$$

Suppose we have density of states g_2 and g_1 .

$$N_2 \propto g_2 \exp(-\beta E_2)$$

$$N_1 \propto g_1 \exp(-\beta E_1)$$

$$p(\nu) = (8\pi h \nu^2 / c^3) (1 / (\exp(h\nu/kT) - 1))$$

$$N_2/N_1 = (g_2/g_1) \exp(-\beta(E_2 - E_1))$$

So we can write,

$$B_{21}p(\nu) + A_{21} = (N_1/N_2) \beta_{12} p_0$$

$$A_{21} = ((N_1/N_2) \beta_{12} - B_{21}) p_0$$

$$A_{21} / ((N_1/N_2) \beta_{12} - B_{21}) = p_0$$

$$A_{21} / ((g_1/g_2) B_{12} \exp(\beta h\nu) - B_{21}) = p(\nu)$$

$$(8\pi h \nu^2 / c^3) (1 / (\exp(h\nu/kT) - 1)) = A_{21} / ((g_1/g_2) B_{12} \exp(\beta h\nu) - B_{21})$$

Therefore,

$$(g_1/g_2) B_{12} = B_{21}$$

$$p(\nu) = A_{21} / (B_{21} (\exp(\beta h\nu) - 1))$$

Therefore,

$$A_{21} / B_{21} = 8\pi h \nu^3 / c^3$$

Therefore,

$$A_{21} = (8\pi h \nu^3 / c^3) B_{21}$$

Remember, $c = \lambda \nu$, hence,

$$A_{21} = (8\pi h / \lambda^3) B_{21}$$

For microwaves the spontaneous emission is negligible. For visible spectra, $B_{21}p(\nu) \gg A_{21}$ for the lasing state.

20/04/1999

Now we rewrite,

$$W_{21} = B_{21}p(\nu) = (A_{21}c^3 / 8\pi h \nu^3) p(\nu)$$

A_{21} is known as the spontaneous emission rate, we write,

$$1/T_{\text{spont}} = A_{21}$$

Experimentalists would say T_{spont} is the spontaneous radiative decay time.

$$W_{21} = c^3 \rho(\nu) / 8\pi h \nu^3 T_{\text{spont}}$$

This is written with an additional term, with the **line shapefunction**,

$$W_{21} = (c^3 \rho(\nu) / 8\pi h \nu^3 T_{\text{spont}}) a(\nu)$$

Where,

$$\text{Integ } (+\text{infin}, -\text{infin}) \{ \rho(\nu) d\nu \} = 1$$

Threshold conduction in a LASER

Consider two energy levels with N_2 electrons and N_1 electrons in them, (N_2 in the upper course). The following relation can be used,

$$c \cdot \rho(\nu) = I(\nu)$$

Where I is the intensity of the beam. ρ is the density of the beam, so in unit time $c \cdot \rho$ is the energy which has flowed through our system. Hence,

$$W_{21} = c^2 I(\nu) a(\nu) / 8\pi h \nu^3 T_{\text{spont}}$$

Now consider,

$$\begin{aligned} dI/dz &= (W_{21}N_2 - W_{12}N_1)h\nu \\ dI/dz &= (N_2 - N_1(W_{12}/W_{21}))W_{21}h\nu \\ W_{12}/W_{21} &= \rho(\nu)\beta_{12}/\rho(\nu)\beta_{21} \\ W_{12}/W_{21} &= \beta_{12}/\beta_{21} \\ &= g_2/g_1 \end{aligned}$$

$$dI/dz = (N_2 - (g_2/g_1)N_1)W_{21}h\nu$$

$$dI/dz = (N_2 - (g_2/g_1)N_1)(c^2 I(\nu) a(\nu) / 8\pi h \nu^3 T_{\text{spont}}) I(\nu)$$

$$dI/dT = y(\nu)I(\nu)$$

$$y(\nu) = \text{'gain'} = (N_2 - (g_2/g_1)N_1)(c^2 I(\nu) a(\nu) / 8\pi h \nu^3 T_{\text{spont}})$$

$$dI/dz=yI$$

$$dI/I=ydz$$

$$\begin{aligned} \ln(I) &= yz + c \\ I &= \exp(yz) \exp(c) = I_0 \exp(yz) \end{aligned}$$

We need to prove that $g_2 = g_1$. If $g_2 = g_1$,

$$y = N_2(g_2/g_1)N_1(c^2 I_0) a(\nu) / 8\pi h \nu^3 T_{\text{spont}}$$

Therefore,

$$y \Rightarrow 0 \text{ if } N_2 \Rightarrow N_1$$

This is population inversion. Recall our 3D relation,

$$\begin{aligned} dN &= (dx/h)(dy/h)(dz/h) dp_x dp_y dp_z \\ &\propto dp_x dp_y dp_z \\ &\propto 4\pi p^2 dp \end{aligned}$$

$$\begin{aligned} p^2 &= p_x^2 + p_y^2 + p_z^2 \\ p &= h/\lambda = \hbar k \end{aligned}$$

$$dN \propto k^2 dk$$

Solid State LASERS

Population inversion is achieved by passing an electric current through the system. First let us look at a bulk 3D solid state laser.

Take a semiconductor, one side n-type the other side p-type, and degenerately dope it. This means that energy levels will spread out,

fig 1

From this we would expect at best to get a photon emitted when the electron transits the band gap, this will not always happen, so it is not 100% efficient. Such lasers are roughly 70% efficient.

fig 2

Solid state lasers are important because they operate at voltages compatible with integrated circuits and can be monolithically integrated with them using photo-lithographic processes used in integrated circuits.

Next we return to an earlier step, we need to determine N_1 and N_2 . Consider the depletion zone of a semiconductor. It has a thickness t and cross-sectional area A . Suppose we put in a total number of N

dopants. So the number of dopants per unit volume is,

$$N/At$$

If the lifetime of (minority) charge carriers is T , then it can be shown that,

$$t=(DT)^{1/2}$$

Where D is a diffusion constant. The charge carriers obey Fermi-Dirac statistics, so we can write,

$$N_2=(N/At)f(E_2)$$

$$N_1=(N/At)f(E_1)$$

Where f is a fermi-dirac function for the conduction and valence bands, indicated by the subscript. So recall that,

$$y(v)=(N_2(g_2/g_1)N_1)(c^2/h^3) \alpha(v) / 8\pi h v^3 T_{spont}$$

$$y=(N/At)(f(E_2)-f(E_1))(c^2/h^3) \alpha(v) / 8\pi h v^3 T_{spont}$$

We now need to change this into a form which has experimental measurements. Let us consider the electric current, I . I/e =number of electrons per unit time multiplied by charge. Hence, I/e =number of electrons per unit time flowing through the system. For semiconductors this is the number of electrons that cross the bandgap per unit time. If N is the total number of electrons, then I/e is also N times the probability that an electron crosses the band gap. So we can write,

$$I/e=N(P_R+P_{NR})$$

Where P_R is the radiative rate and P_{NR} is the non-radiative rate (Auger, multi-phonon emission). We now write,

$$I/e=N(P_R+P_{NR})(P_R/P_R)$$

$$=N((P_R+P_{NR})/P_R)(1/T_{spont})$$

$$N/T_{spont}=(I/e).(P_R/(P_R+P_{NR}))=(I/e)n_p$$

Where n_p is the radiative efficiency of the system.

$$n_p=P_R/(P_R+P_{NR})$$

$$y=(1/At)(f_c(v)+f_v(v))(c^2/h^3) \alpha(v) / 8\pi h v^3 e (I/e)$$

$$y=(1/At)(f_c(v)+f_v(v))(c^2/h^3) \alpha(v) / 8\pi h v^3 I$$

$(f_c(v)+f_v(v))$ is called the **fill factor** (it's maximum value is one). So $y_{threshold}$ is required to before lasing begins.

$$I_{\text{threshold}} = \frac{A t}{(f_c(v) + f_v(v)) \cdot (1/a)}$$

In 3D structures, A is in mm² and t is 10000 Å. In 2D structures A is in mm² and t is 100 Å.

Now let us consider the threshold conditions. If we have a depletion zone of width 1 μm. The wavelength of light is often larger than this, so we would have a source of loss by means of the n and p regions, we also have gain in the depletion zone. Now consider that if we have two mirrors separated by a space L, and photons reflect between the two and amplify each other (imagine the depletion zone barriers are like mirrors!)

$$L\Gamma = a_a L\Gamma + a_c(1 - L\Gamma) + (1/L)\ln(1/e)$$

LΓ is the fraction of lasing mode confined to the active region. The Ln term is the distributed mirror loss term. a_n and a_c are the optical losses,

$$R = (R_1 R_2)^{1/2}$$

R₁ is the reflectivity at mirror one and R₂ is the reflectivity at mirror 2.

Now consider a wave of unit power, which travels a distance 2L (return trip from one mirror the other). When it returns its power will be,

$$1 * R_1 * R_2 * \exp(g - a)2L$$

g is the gain and a is the loss, both per unit length. Threshold condition is when,

$$1 = R_1 * R_2 * \exp((g - a)2L)$$

Now take logs of both sides,

$$0 = \ln(R_1 R_2) + 2L(g - a)$$

now write,

$$g = a - (1/2L) \cdot \ln(R_1 R_2)$$

also note,

$$\ln(1/R_1 R_2) = \ln(1) - \ln(R_1 R_2) = -\ln(R_1 R_2)$$

$$\begin{aligned} g &= a + (1/L) \cdot \ln((1/R_1 R_2)^{1/2}) \\ &= a + (1/L) \cdot \ln(1/(R_1 R_2)^{1/2}) \end{aligned}$$

So recall from above,

$$L\Gamma_{cl} = L\Gamma_{cl} + (1 - L\Gamma_{cl})\alpha_c + (1/L) \cdot \ln(1/(R_1 R_2)^{1/2})$$

α_c is the loss in the cladding layer and α_a is the loss in the active region.

Now we wish to find out what the power output is.

$$P = ((I - I_{th})/e) h\nu \cdot n$$

$$P = ((I - I_{th})/e) h\nu \cdot n \cdot ((1/L) \cdot \ln(R) \alpha_a L\Gamma_{cl} + \alpha_a L\Gamma_{cl} + (1/L) \cdot \ln(1/R))$$

27/04/1999

Ultra low pressure

This involves sending atoms onto a substrate. The Mean free path between collisions is large (this is referred to as molecular flow). The general process this falls under is called **Molecular Beam Epitaxy** (MBE). The processes in MBE are well understood. MBE allows us to grow monolayer by monolayer, which allows us to make atomically abrupt interfaces, both of these are advantageous. The monolayer transition region is small (such regions are termed *interface roughness*). The disadvantage with this is that it is very expensive. Another disadvantage is that the growth rates are very slow, typically one Angstrom every 10 seconds. See fig 1 for how different pressures are used for different materials. Because of the nature of this process, there is no boundary layer formed.

fig 1

Vapour Phase Epitaxy

A mixture of gases flow across a substrate and interact with one another. The flow is viscous, and a boundary layer is formed. The chemical processes involved are poorly understood. The interfaces are not (in general) as atomically abrupt as in MBE. The main advantage is that it is very cheap in relation to MBE.

Now consider we were to grow a super lattice, with a potential as shown in fig 2

fig 2

The period of the structure is d ($\sim 100\text{\AA}$). The period of bulk-interatomic spacing is (1\AA) . The energy bands are split into what are known as **Brillooinzones**. The Brillouin zone in the bulk is written as,

$$-\pi/a \leq k \leq \pi/a$$

So for our superlattice,

$$-\pi/d \leq k \leq \pi/d$$

fig 3

Isaaki argued the following, a charge carrier can get trapped in a Brillouin zone, on the E-K curve of figure 3. It's momentum is then periodically reversed/reflected. This will give us an AC current from applying a DC voltage. This has not been observed yet, but the search continues ! Such a trapped oscillating charge, is known as a **BlochOscillator**

Consider the energy relation,

$$E = \hbar^2 k^2 / 2m$$

$$dE/dk = \hbar^2 k / m = \hbar (\hbar k / m) = \hbar (p/m) = \hbar v$$

$$v = (1/\hbar) (dE/dk)$$

$$F \cdot e = dp/dt$$

(field strength)

$$= \hbar (dk/dt)$$

$$eF/\hbar = dk/dt$$

$$\text{Acc'n.} = dv/dt = dv/dk * dk/dt$$

$$v = (1/\hbar) (dE/dk)$$

$$dk/dt = eF/\hbar$$

$$a = dv/dt = (dv/dk)(dk/dt)$$

$$= (1/\hbar^2) eF (d^2E/dk^2)$$

At the point of inflection on our E-k plot, as the momentum increases the acceleration reduces, that indicates negative differential resistance! This was another prediction from Isaaki that has not been realised yet.

Douisle Barrier Tunneling Diodes

Consider a conduction band function, with two barriers, the barriers are 10 Å wide and the spacing between them is 50 - 100 Å. To either side of the pair of barriers, we degenerately dope the substance.

fig 4

The electrons still cannot get past the barriers. Suppose we have an electron trapped between the barriers and we make the barriers infinitely high. If the electron makes a standing wave, we have,

$$\begin{aligned}d &= \lambda / 2 \\ \lambda &= 2d \\ p &= h / \lambda\end{aligned}$$

$$\begin{aligned}E &= p^2 / 2m = h^2 / 2m\lambda^2 \\ &= h^2 / 8md^2\end{aligned}$$

This is called a resonance state, since the electron makes a resonant curve in the *well*.

For electrons in a bulk metal structure, we have the fermi energy,

$$E_f = \hbar^2 k^2 / 2m$$

If we have the same barriers structure in our metal as described above. There is a resonant energy level in between the barriers. If we apply a voltage we rise the fermi level, once the fermi level is at the same height as the resonant level, they can tunnel through the barrier. On the outside of the barriers we have a 3D system and we have a *fermi-sphere* of electrons which have the same energy

fig 5

n-i-p-i Structures

fig 6

This is simply structure doped so that we have n type, an insulator, p type and another insulator. These structures give rise to Type II superlattices. Their use is good for non-linear optics. Suppose that the concentration of n types is the same as that for p types, this is called self-compensated. We have ionized donors and ionized acceptors. Consider the potential profile.

fig 7

If we have a square well and choose our origin to be in the centre a charge carrier would see the same potential either side. We could also say for at that point,

$$\mathbf{dV/dx=0 \text{ (at } x=0)}$$

If we then solve for Poisson's equation,

$$\mathbf{\partial^2V/\partial x^2=4\pi e^2 n \epsilon}$$

n is the dopant concentration.

$$\mathbf{\partial V/\partial x=(4\pi e^2 n \epsilon)x + C}$$

$$\mathbf{\partial V/\partial x=0 \text{ at } x=0 \text{ therefore } C=0}$$

$$\mathbf{\partial V/\partial x=(4\pi e^2 n \epsilon)x}$$

$$\mathbf{V=(4\pi e^2 n x^2/\epsilon)+C_2}$$

$$\mathbf{V=0 \text{ at } x=0 \text{ hence } C_2=0}$$

We now work out what V is at $x=d/2$ (d is the width of the well).

$$\mathbf{V=(4\pi e^2 n/\epsilon)(d/2)^2}$$

The depth of the well depends on n and d.

the end (I think)