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NUCLEAR PHYSICS

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Reading

Nuclear & particle physics, W.S.C. Williams, Oxford (1992)

A brief history

1896
Bequerels discovered radioactivity.

1913
Curie investigated radioactivity.
Geiger & Marsden experimented with Rutherford scattering.
Rutherford interpreted the results of Geiger & Marsden (alpha particle probing).

1919
Rutherford split the atom (bombarding nuclei with alpha particles to release a proton.

1930
Chadwick discovered the Neutron.

1930+
Hahn investigated nuclear reactions, particularly neutron reactions.

1939
Hotan & Strassman discovered nuclear fission.
Meither & "" realised the potential of the chain reaction.

1942
Fermi started the first controlled chaing reaction.
Atomic and Hydrogen (fusion) bombs wefe developed.

Since the 1950's most work has been put into investigating nuclear structure, also work on transuranic elements, artificial radioactivity, radioactive dating, nuclear medicine and nuclear power.

Nuclear Masses

Nuclear masses were first calculated using Bambridge's mass spectrometer.

Fig 1

In the velocity filter,

$$F_b=BQv$$
$$F_e=QE$$

<--(+)-->

For positive ions that are undeflected,

$$BQv=QE$$
$$v=E/B$$

In the velocity selector,

$$F_b=BQv=Mv^2/R$$

Fig 2

$$BQR=Mv=ME/B$$
$$M=B^2QR/E$$

Results from this apparatus found the following atomic masses,

Hydrogen=1

Helium=4

Nitrogen=14

Oxygen=16

It was also found that particular elements had **isotopes**and thier relative abundances were calculated. For example, Neon has two isotopes, ^{22}Ne=90% and ^{20}Ne=10%.

Isotopic Mass

It was internationally decided that isoptopic masses would be measured relative to the Carbon-12 isotope. This was because of Carbon-12's availability and isotopic abundance (~99%). By definition ^{12}C has 12 atomic mass units (amu or u). Also by definition, 12g=12amu*N_a (where N_a is Avandagro's number $6.023*10^{23}$). Other isotopic masses measured on the ^{12}C scale are,

^{1}H=1.007825u

^{16}O=15.9949u

Atomic Mass

^{16}O	15.99941	99.768%

^{17}O	16.99910	00.037%
^{18}O	17.99920	00.204%

Which gives the average atomic mass for Oxygen of 15.9949u.

Nuclear structure

-Relative abundace of neutons and protons in nuclei, see Figures NP1 and NP2.

Fig NP1
Fig NP2

Nuclear stability of heavier nuclei.

In general for light nuclei, N=Z, where N is the neutron number and Z is the proton number. For heavier nuclei however, in general, N>Z. Nuclear stability is determined by the coulomb force at long ranges and nuclear forces (strong and weak) at short ranges. Therefore for heavy nuclei coulombic repulsion increases and the nuclear attraction remains the same. So increased stability occurs for a greater number of neutrons.

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Nuclear binding energy

The mass of a nucleus is less than it's component parts. Hence we can write,

$$M(Z,A)=ZM_pC^2+NM_NC^2-B$$

Where B is the binding energy. It can be described by the **liquid drop** model of the nucleus. The idea behind this model is that the nucleus has constant density like a drop of liquid. Also, there is a surface tension effect because surface nucleons are otherwise less tightly bound (in analogy only!). The binding energy in this model is given by,

$$B(Z,A)=a_vA-a_sA^{2/3}-a_cZ^2/A^{1/3}$$

Where a_vA is the volume term, $a_sA^{2/3}$ is the surface term and $a_cZ^2/A^{1/3}$ is the coulomb term. The latter two terms serve to reduced the value of B.

Volume term.
The total energy is proportional to the value of A and hence B/A is approximatley constant for all nucleides.

Surface term.
The surface nucleons are less tightly bounbed because they

have less neighbours. The reduction in the binding energy is proportional to the surface area of the nucleus. If the nucleus has a radius R then the binding energy reduction is also proportional to R^2. We can easily say that, mass=vol*density. We know that density is constant and that volume is proportional to R^3 and if the mass is proportional to the atomic number (A). Therefore we can say $R^3\ \alpha\ A$, $R=r_oA^{1/3}$. Therefore the surface tension term is proportional to R^2 and proportional to $A^{2/3}$ (described by the liquid drop model).

Coulomb term.

If we have a charge density þ and an element of charge, the element of charge can be calculated as,

$$\mathbf{q=4pir^2dr\text{þ}}$$

The charge in the volume (sphere of radius r) is,

$$\mathbf{Q=(4/3)pir^3\text{þ}}$$

So the total energy is written as,

$$\mathbf{E=Integ\ (R,0)[(((4/3)pir^3\text{þ})(4pir^2\text{þ})dr)/(4piE_or)]}$$
$$\mathbf{=(16pi^2\text{þ}^2R^5)/(15(4piE_o))}$$

but,

$$\mathbf{Q=(4pi/3)R^3\text{þ}}$$

Therefore, the potential energy is,

$$\mathbf{V=3Q^2/(5(4piE_o)R)\quad \alpha \quad Z^2/A^{1/3}}$$

Hence we write for the binding energy,

$$\mathbf{B=a_vA-a_sA^{2/3}-a_cZ^2/A^{1/3}}$$

We can see that this is not the whole picture because this suggests that most stable atoms would have only neutrons, but there is no such thing! It was found that there were two more terms to be added to the binding energy term to account for this difficiency in the model.

The first term to be added was the **Asymmetric energy term**. For this we consider nucleons to be like particles in a potential well, which we call the *nuclear potential*. Because of the extremely small dimensions the nuclear potential energy is quantised. For each energy level in the nucleus it was found that there could be two protons and two neutrons each with opposite spins on each energy level. Hence, protons and neutrons are equally numbered so Z is apporximately the same as N.

fig 1

We now look at the additional energy for a nucleus with Z not equal to N, for 1, 2, 3, 4, 5, 6, 7 extra neutrons. If we conisder dE, the energy gap between nuclear energy levels. The additional energy is then some multiple of dE. This is because for a given energy level to have more neutrons we need to push a proton up to a higher energy level. The respective additional multiples of dE is then, 1, 1, 3, 3, 5, 5, 7 (*dE). Hence the total energy is summed as, 1, 2, 5, 8, 13, 25 (*dE). The total energy can generally be expressed as,

$$E=(N-Z)^2 dE/8$$

We find that dE decreases up the nucleus such that dE α 1/A

$$E=(N-Z)^2/8A$$

The second term to be added to the binding energy was the **Paring term**. There are several numerical combinations of combinations of Z and N

A is even,
- Z even and N even : these are very stable.
- Z odd and N odd : These are few becuase they are unstable.

A is odd,
- Z even and N odd
- Z odd and N even
- Both of these have intermediate stability.

fig 2

The paring term, $\delta(Z,A)=0$ for the even-odd atom, $=+a_p/A^{1/2}$ for the even-even atom and $=-ap/A^{1/2}$ for the odd-odd atom.

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The overal binding energy can now be written as,

$$B=a_vA-a_sA^{2/3}-a_cZ^2/A^{1/3}-a_A(A-2Z)^2/A\pm a_p/A^{1/2}$$

Without the pairing term, we would only have one stable isotope.

Further evidence of the pairing term.

(Both of the following figures come from observation.)

fig 1

From figure 1 we can see we only have one stable isotope.

fig 2

So, pairing leads to extra stability of nuclei and leads to isotopes etc.

The nuclear cross section.

The nuclear cross section is a measure of the *effective* size of the nucleus. If we consider a collision between a nucleus and another particle, consider that both have wave functions. The incoming particle would not neccessarily have to make a direct hit to peturb the nucleus. So the effective size is the area within which a reaction would take place.

fig 3

From figure 3 the ratio of incoming particles to collisions is,

N_o/N=ònAx/A=ònx

Therefore,

ò=(1/nx)(N_o/N) (m^2)

This is the nuclear cross section. However this is in the case of particles exiting in the same direction they entered, we now consider the **differential nuclear cross section** (this describes a more scattering-like process). We now consider system where N particles incident over ò and N_o leave with the solid angle d-omega.
Now,

No=nxNò
dN_o/domega=nxN(dò/domega)
dò/domega=(1/nxN)(dN_o/domega)
(The differential cross section)

Rutherford scattering

fig 4

Using the schematic set up shown in fig 4, Rutherford measured the differential cross section from scattering alpha particle through a thin film of gold atoms.

See "Rutherford sccattering formula" sheet

Rutheford assumes a massive nucleus, he also assumed the alpha particles were deflected by the coulomb force, he then showed,

dò/domega=(Zze^2/16piE_0T)cosec^4(theta/2)

fig 5

Rutherfords data fitted well to his model except for aluminum foil for scattering at back angles (ie theta>90). The number of back scattering was less than preedicted, he realisd this was because the alpha particle just touches the nucleus (Al was the lightest, and hence smallest, of the materials he used).

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Size of the nucleus from Ruterford scattering

Rutherford found that there were insufficient alpha particles being back scattered from what was predicted. If we consider the incident energy, it is transfered to potential energy when thealpha particle just touches the nucleus.

E=2Ze^2/4piE_0R

R is the nuclear radius. E is 4e6*1.6e-19=6.4E-13 Joules which gives R the value, R=0.9e-14.

Nuclear Size

In the present day nuclea size is measured by using high energy electrons, typically 200 MeV. These have a De Broglies wavelength which is comparable to the size of the nucleus. Recall the relation,

$E^2=p^2c^2+m_o^2c^4$
&
lambda=h/p

and if the $m_o^2c^4$ term is 0.5 MeV we can make the approximation,

E=pc

So we can then say,

lambda=h/p=hc/E
=(6.6e-34*3e8)/(200e6*1.6e-19)
=6.1875E-15

fig 1

In figure one, in the magentic field, where deflected by the right amount for elastic scattering at an angle theta.

So to measure dò/dw, we plot dò/domega against theta.

fig 2

Without proof, the diffential cross section for scattering is given by,

$$dò/dw=(dò/dw)|_{Rutherfod}\times (1-(v^2/c^2)\cos(theta)) \; . \; |F(theta)|^2$$

Where,

F(theta)=(1/Ʒ.Integ() {þ(r).exp(iqr).dV}

(Where þ(r) is the charge density).
F(theta) adds up the Rutherford scattering term from each element of charge in the nucleus given tat interefence occurs.

fig 3

The factor of q is,

q=2psin(theta/2)
fig 4

The momentum transfered to nucleus.

If the density is constant the form factor has the form of interference peaks wit respect to q. See fig 5.

fig 5

The final result is is that the charge density remains consstant for all nuclei but the surface densit drop quickly over a short range.

fig 6

<u>The Shell model of the nucleus.</u>

The liquid drop model did not predict everything about the nucleus. The shell model allows us to predict more. We can predict; 1) the spins of the nucleus, 2) Parity of the nucleus and 3) Magic numbers. Magic numbers give values of A,Z and N when nuclei are more stable. Magic numbers in this subject are; 2, 8, 20, 28, 50, 82 and 126.

<u>Evidence for Magic Numbers</u>
1) Binding energy per nucleon is great than that or neighbouring nuclei (see sheet).
2) The binding energy of Helium (A=4, Z=2, N=2) and oxygen (A=16, Z=8, N=8) is very high (these are known as doubly magic, Z=N).

3) The abundance of magic nuclei is greater than that of eighbouring nuclei. eg, Pb(A=208, Z=82, N=126) is very abundant.
4) When the Z is magic there are many stable isotopes. $_{50}$Sn has ten stable isotopes.
5) Likewise with 4, when N is magic there are many stable isotones.

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The shell model assumes that the nucleons are held in a potential well, formed by nuclear force, of a form,

fig 1

The nucleons have quantized energy states in this nuclear potential. By solving the Schrödinger equation for this potential we find the energy for each nucleon is,

E_{nl}=(2n+l-½)*hbar*w

Where n is the principal quantum number, it can have any value (n=1,...). l is the orbital angular momentum quantum number, it can also take any value (l=0,...).

The lowest energy occurs when n=1 and l=0, which gives **E=(3/2)hbar*w**. The next lowest energy level occurs when n=l=1, this gives **E=(5/2)hbar*w**. The next lowest energy level is now **E=(7/2)hbar*w** which occurs when n=2 and l=0 or when n=1 and l=2.

fig 2

Magic numbers occur when nucleus is extra stable, ie when shells are full.

Higher energy levels do not fit the magic numbers, in order to get achieve a fit we introduce **spin orbit coupling**, which is described by the total angular momentum quantum number, j.

j=l±s=l±½

For nucleons, s=½.

For the shells,

	s	p	d	f	g
l=	0	1	2	3	4
j=	½	½,1.5	2.5,1.5	3.5,2.5	4.5,7.2
No of nuc's in level					

j 2 2,4 6,4 8,6 10,8
=(2j+1)
$_{mJ}$=±j,..,0

Hence all the magic numbers are correctly predicted.

Prediction of nuclear spins

Nuclear spin was predicted from the shell model.

Spin=Sum of all angular momentum terms

Assume for level j there are (2j+1) particles, with values of +m_J and -m_J. For an even numbere of nucleons in level j, the net angular momentum is zero.

eg,
$^{4}_{2}He$, Z=2 N=2 Therefore I=0
$^{16}_{8}O$, Z=8 N=8 Therefore I=0

So for all even-even nnuclei, I=0, as is found. For Odd-Even nuclei assume I is justt that of the last nucleon.

eg,
$^{17}_{8}O$ Z=8,N=9. The protons are paired off so do not contribute to the angula momentum. The only contribution of the angular momentum will be from the ninth neutron. Therefore the ninth neuton is in the d(5/2) state (see sheet) with total angular momentum **=(5/2)hbar**. Therefore,

I=(5/2)hbar

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Parity

This is related to the way the particle's waev function changes when it is reflected in the origin.

fig 1

All wave function have the property,

Psi(r)=±Psi(-r)

Where when we have **+Psi(r)** we say it has positive parity and when we have **-Psi(r)** we say it has negative parity. Parity depends on whether the particle has odd or even angular momentum.

P=(-1)^l

Where **l** is the angular momentum.

Parity of the Nucleus depends on the parity of all the nucleons. Since,

Psi(nucleus)=Psi(nucleon 1) * Psi(nucleon 2) *...
Psi(nucleus)=SumProd (-1)^l

Therefore for a A is even, **P(nucleus)=+1** For A is odd, **P(nucleus)=P(last nucleon)**
For example, $^{16}_{8}O$ parity, P=+1. $^{17}_{8}O$ parity is determined by the last neutron which is in the $d_{(5/2)}$ state for which l=2, so $P=(-1)^2=+1$. $^{7}_{3}Li$, the last neutron is in the $P_{(3/2)}$, where l=1, so P=-1.

Excited states and spins of atoms

Consider $^{17}_{8}O$,

fig 2

From figure 2, the ground state so $I=½^+$ in the excited state $I=5/2^+$

Nuclear reactions

Rutherford

Rutherford observed the emission of alpha particles from a sample of Polonium in a chamber filled with some gas (O_2, Nu2, CO_2, air, etc).

fig 3

He observed how far the alpha particle travelled through each gas. He did this by watching for flashes on a zinc sulphide screen at the end of the chamber as he drew the polonium sample away from the window.

He found that the number of counts was fairly constant, and then suddenly drops as he drew the sample away. For air and Nitrogen he found that few particles had a range of 40 cm.

fig 4

He applied a magnetic field to the chamber as still observed some counts, so he deduced that he was observing protons as well as alpha particles. He assumed the following nuclear reaction,

$$^{4}_{2}He + ^{14}_{7}N \rightarrow ^{13}_{6}C + ^{1}_{1}H + ^{4}_{2}He$$

Further to Rutherford's work, Blackett used a cloud chamber to track the alpha particles.

fig 5

From his results he deduced,

$$^{4}_{2}He + ^{14}_{7}N \rightarrow ^{1}_{1}H + ^{17}_{8}O$$

<u>Nuclear cross section</u>

Resuts from these nuclear reaction experiments also provide a means to deduce the nuclear crcoss section.

fig 6

Then,

$$N_o/N=\grave{o}nAx/A=\grave{o}nx$$

Therefore,

$$\grave{o}=(1/nx)(N_o/N)$$

But,

$$n=(N_a/22.4)*10^3$$

The 22.4 is the specific volume of gas at STP and the 10^3 is a conversion factor for SI units. So,

$$\grave{o}= (22.4/(6e23*1e3*0.2))*(8/400000)=0.22e-24\ cm^2$$
$$=0.22\ barns$$

<u>Neutrons</u>

Most materials when bombarded wih alpha particles produce protons, with the exceptions of B and Be. It was found that when a large sample of Boron was bombarded with alpha particles, penetrating radiation was observed to be emitted. It was thought at the time to be gamma rays. After passing this radiation through parathin wax, that protons were released (energy of 5MeV). It was considered the alledged gamma rays were causing compton scattering in the wax which released th proton. But for the proton to have 5MeV on release qould require th incoming photon to have 50MeV of energy, this would happen if the following was true,

$$^{4}_{2}He + ^{11}_{5}B \rightarrow ^{15}_{7}N + y$$

But the maximum energy this photon could have was 16 MeV, so this overall process was not possibe. Chadwick proposed that the radiation was a neutral particle. THe "Neutron" which would have the mass of a proton. Until then nuclear structure, it was thought that the mass number was the same as the proton number, ie A protons and (A-Z) electrons. Now we use, Z protons and (A-Z) neutrons.

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Nuclear reactions

eg, Rutherford's discovery: $^4He + ^{14}_7N \rightarrow ^1_1H + ^{17}_8O$

Generally we have at low energy (of incident particle) we have,

$$\mathbf{a + X \rightarrow b + Y}$$

Where a is the incident nucleus, X is the target nucleus, b is a light nucleus and Y is a heavy nucleus. b and Y depend on the energy of a, and severa different reactions may take place. Another notation for such reaction is X(a,b)Y.

Examples:

- (α,p) Rutherford's $^{14}N(\alpha,p)^{17}O$
- (p,α) Inverse, $^{16}O(p,\alpha)^{13}N$
- (d,p) Stripping, $^{14}N(d,p)^{15}N$
- (p,d) Pick up, $^{14}N(p,d)^{13}N$
- (α,n) Chadwick's $^9B(\alpha,n)^{12}C$

Q-Value

A small nucleus of mass m is incident on a large nucleus of mass M. After, two nuclei of mass m' and M' are created.

$$\mathbf{Q=[(m+M)-(m'+M')]c^2}$$

If Q>0 the reaction proceeds for all energies of the incident nuclei, the threshold energy is then zero. If this is so the reaction is said to be **exoergic**

If Q<0 then the threshold energy is greater than |Q| (not not equal to, because momentum would not be conserved). Such a reaction is known as an **endoergic**reaction.

Threshold energy

fig 1

In the center of mass frame of reference,

$$mv_c=Mv_o$$

Also,

$$v_c=v-v_o$$

Therefore,

$$m(v-v_o)=Mv_o$$
$$v_o=mv/(M+m)$$
$$v_c=v_o*(M/m)=Mv/(M+m)$$

Conservation of energy

(In the center of mass frame of reference)

$$mc^2+T_{m,c}+Mc^2+T_{M,c}=m'c^2+T_{m',c}+M'c^2+T_{M',c}$$

(where $T_{m,c}$ is the kinetic enrgy of particle m in the center of mass frame of reference)

Therefore,

$$Q+T_{m,c}+T_{M,c}=T_{m',c}+T_{M',c}$$

Hence, for the reaction to proceed,

$$Q+T_{m,c}+T_{M,c}=>0 \qquad (1)$$

But,

$$T_{m,c}=½mv_c^2=½m(M^2v^2/(M+m)^2)=T_m(M^2/(M+m)^2) \quad (A)$$
$$T_{M,c}=½Mv_o^2=½(Mm^2v^2/(M+m)^2)=T_m(Mm/(M+m)^2) \quad (B)$$

So we can now relate the kinetic energies in both the lab frame of reference and center of mass frame of reference.

Equation one gives,

$$Q+T_m((M^2/(M+m)^2))+(Mm/(m+M)^2)) => 0$$
$$Q+T_m(M/(M+m)) => 0$$
$$T_m => -Q((m+M)/M)$$

If Q>0, the threshold energy is zero, but if Q<0, the threshold energy is,

|Q|((m+M)/M)

Conservation laws

1. Linear Momentum and Energy
2. Z, A
3. Total spin angular momentum (am) and Parity

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Conservation of Total Angular momentum and Parity

fig 1

In general,. if two colliding nuclei are not hitting each other head on, there is an orbital angular momentum between them, which is,

l.hbar

If they collide head on l=0, for T (KE) at about 10 MeV. l=1 for T approximately 20 MeV. For l=0 we say it is "S wave scattering" and for l=1 we call it "p wave scattering".

Parity

The symmetry of the wave functions when the wave function is reflected in the origin. ie,

Psi(r)=±Psi(-r)

ie, positive or negative parity.

Each nucleus has an intrinsic parity (determined from the shell model). The parity that is associated with the orbital angular momentum of te ttwo nucleus about each otehr when they scatter.

Total Parity = (Product of intrinsic parities) * (Parity of the orbital wave function)

Parity of the orbital wave function
L=(-1)

Total Parity = $P_{T1} * P_{T2} * (-1)$

Example

Consider Berillium and Boron,

$$^{10}_{5}B + ^{4}_{2}B \rightarrow ^{1}_{1}H + ^{13}_{6}C$$

Nuclear spin is I=3,0,½,½ respectively for the above nuclei. Putting in the sign for the parity we get (from the shell model), I=+3,+0,+½,-½. For a low energy collision (T < 10MeV) l is always zero.

Initially,

$$\textbf{Total angular momentum} = 3+0+0=3$$
$$\textbf{Total Parity} = (+1)*(+1)*(-1)^{l=0}=+1$$

Now consider after the reation,

fig 2

The total angular momentum is 3 (still !) and the tota parity is +1. The total spin of two nuclei is 1 for parallel spins and 0 for anti-parallen spins. ie, I = 1 or 0
Therefore,

$$3=I+l$$

Therefore,

$$l=4,3,2$$

Parity

$$(+1)=(+1)*(-1)*(-1)$$

Therefore,

$$l=3$$

Therefore total angular momentum and parity are conserved. Therefore the reaction is said to be allowed.We also find that l=3 in the final state.

Reaction Mechanisms

The reaction mechanism depends largely on the energy of the incident particle. At energies T <= 1 - 10 MeV. If we havae nucleus a incident on nucleus X. Elastic scattering can take place, ie the two simply bounce off each other without any nuclear interactio taking place. The always happens and kinetic energy is conserved. Nucleus a can get inside nucleus X and become part of a **compound nucleus**This is formed in the excited state. A further process is that two new nuclei b and Y are formed wqhen the compound nucleus splits up. This decay occurs in approximately 10^{-14} s.

cf. to typical nuclear time scales. The time for a nucleus to pass another is,

$$\textbf{Time} = \textbf{Diam} / \textbf{vel}$$
$$= 10^{-14}/10^{7}$$
$$= 10^{-21}\ \text{s}$$

<u>Evidence for compound nucleus formation</u>

1) Branching ratios

fig 3

The branching ratio (ie frequency of occurence of products) is the same for both modes of formation. ie Compound nucleus *forgets* how it was formed.

2) Nuclear cross sections

Consider a sheet of Cadmium foil, which is bombarded by neutrons. We use a detector to measure the decrease of N_o (the original number of neutrons) when the foil is put in place (ie N_o-N). We know that,

$$ò=(1/ndx)(N/N_o)$$

fig 4

The resonant peaks in fig 4 correspond to the formation of a compound nucleus. Also the energy (or mass) of the compound nucleus nucleus is measured to an accuracy of the width of the resonance peaks (approx. 0.1 eV). The uncertainty principal tells us how long we have to measure an energy to a particular precsion.

$$dEdt \sim hbar$$
$$0.1*1.6e\text{-}19*dt \sim (1/2pi)*6.6e\text{-}341$$
$$dt \sim 10^{-14}\ \text{s}$$

Therefore the compound nucleus must exist for at least 10^{-14} s.

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Reaction energies greater than 1eV, Direct reactions

For E>1, we have direct reactions, which means the incident nucleus reacts with the target nucleus as a whole. Evidence from this comes from cross section measurements (the familiar set up of particle beam incident on thin film with a detector on the other side of the film). Measuring the cross-section, δ_T, this is made up from the combination of elastic and inelastic collisions.

fig 1

fig 2

Using the uncertainty principal gives us the time of reaction from the presscission that energy is measured.

dtdE ~ hbar

Therefore (if $dE=10^6 * 1.6e10^{-19}$)

$$dt=10^{21} J$$

This the evidence for a direct reaction, this is less time than it takes to cross the nuclear diammeter.

Optical Model of the Nucleus

Because we know that nuclei have wave properties, we can attribute wave / light like models to them too. If we consider the nucleus as a translucent sphere (partly absorbing and partly reflecting) we can have the following system.

fig 3

The absorbed wave represent inelastic scattering and the transmitted wave represent inelastic and elastic scattering. If we consider a a wave function tunneling through a potential well, we make use of a complex potential,

$$-(V_o+iU_o)$$

Where V_o is the elastic collision energy and U_o is the inelastic collision.

The optical model is used to solve the "Gross Structure" problem (see sheet), with V_o=42 MeV and U_o=3 MeV. In this energy range all nuclei appear to be equally translucent. The optical model works when lambda of the incident particle is of the order of the nuclear size.

Higher energies, E~=>50 MeV

Here lambda is less than the nuclear size, incident particles interact with only part of the nucleus.

$$^{1}_{1}H + ^{4}_{2}He \rightarrow ^{1}H + ^{1}n + ^{3}He$$

fig 4

The ^{3}He is said to be a *spectator*, it does not paarticipate in the reaction.

Nuclear Energy Levels

Nuclear energy levels are deduced from relatively low energy nucelar reactions.

$$^{1}H + ^{7}Li \rightarrow (^{8}Bi)^{*}$$
$$(^{8}Bi)^{*} \rightarrow 2^{4}He$$
$$(^{8}Bi)^{*} \rightarrow n + ^{7}Be$$
$$(^{8}Bi)^{*} \rightarrow 2^{4}He + y$$

The above are the possible decay processes for ^{8}Bi. If we bomard ^{7}Li with ^{1}H and then use a detector that can distinguish between alpha particles, neutrons and gamma rays. We then plot the inelastic cross section againt the lab-frame-of-reference ^{1}H kinetic energy, we get the following,

fig 5

In the centre of mass frame of reference, there is zero kinetic energy.

$$m_H + T_H + m_{Li} + T_{Li} = m_{Be} + E^{*}$$

Where E^{*} is the excited state energy for ^{8}Be.

$$E^{*} = m_H + T_H + m_{Li} + T_{Li} - m_{Be}$$

From equations A and B (see threshold energy),

$$T_H = T_p(lab) M_{Li}^2 / (M_{Li} + M_p)^2$$
$$T_{Li} = T_p(lab) m_H m_{Li} / (M_{Li} + M_p)^2$$

So T_H(lab) is measured, this gives T_H and T_{Li}, gives E^*.

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Now let us consider an energy level diagram of the compound nucleus.

fig 1

The decay route is determined by the excited state which the compound nucleus goes into at it's creation. The reason why each decay route is followed is due to conservation laws lf **spin** and **parity**.

Spin parity of 2$^{4}_{2}$He

For ^{4}He, I=0

fig 2

A possible wave function for the two alpha particles are,

$$Psi_a(1)*Psi_b(2)$$

Another possible wave function is,

$$Psi_b(1)*Psi_a(2)$$

These two wave functions are different, but the alpha particles are indistinguishable particles, so interchanging the co-ordinates of the particles leaves the system invariable. So, the correct wave function to choose is,

$$Psi=Psi_a(1)Psi_b(2)\pm Psi_b(1)Psi_a(2)$$

Change the particles over by applying the parity operator (P),

$$P\ Psi = Psi_a(2)Psi_b(1)\pm Psi_b(2)Psi_a(1)$$
$$= \pm\ Psi$$

So _Psi_^{2} is unchanged. For bosons, Psi has to be symmetric, for fermions Psi has to be anti-symmetric (it is the anti-symmetric that leads to the Pauli exclusion principal). Alpha particles are bosons (I=0), and Psi is symmetric, ie $Psi_a(1)Psi_b(2)+Psi_a(2)Psi_b(1)$, therefore P is positive. With the two alpha particles together they have an angular momentum of l and the parity is $P=(-1)^l=+1$. Therefore l must be an even value (0,2,4,...).

fig 4

19.9 level	2 alpha's directly with l=2, P positive.
19.19 level	n+^{7}Be, P not conserved.
17.64 level	Emits y, dI=±1, dP=+1, then 2 alpha's with I and P conserved

The End !!! :-)