David.R.Gilson

ASTROPHYSICS

12/02/1999

Contents

Physics and chemistry of celestrial bodies (and spaces between).

Terestrial physics aplied to extra-terestrial material.

Problem - difficult to do *active* experiments.

Rely upon observation.

We can only *see* the photons (or lack of them).

Visible Astronomy

In this area of observation we observe wavelengths between 400 to 800 nm. There are many observed stars in our galaxy (about 10^{11}), and our galaxy is one out of 10^{10} observed galaxies.

The nearest star to the earth is the sun (at the speed of light is 8.3 minutes away). The next nearest star is Proxima centauri is 4.3 years away. Canis major (sirius - orion's dog) is almost 10 years away.

Not all stars are the same, we can only obtain information about them by analysing thier energy.

Glossary

1) Astronomical unit (AU) is the average distance from the sun to te earth. 1 AU = 1.496×10^{8} km.

2) Light year (ly) is the distance travelled by light in one year. 1ly=9.46×10^{12} km

3) Parsec (pc) is the distance which 1 AU subtends an angle of 1 arc second. 1 pc = 3.09×10^{13} km.

Creation of stars

Consider a spherical body of uniform density, mass M and radius R. Now divide the sphere into an inner sphere and outer shell ofequal mass, M/2. So the radius of the inner sphere is,

$$r=R/2^{(1/3)}=0.79R$$

We need to consider the gravitational froces between the inner and outer components. The surface of the inner shpere is **$A=4pi(0.79*R)^2$** The inward gravitational force onto the surface of the inner sphere is,

$$F_o=G(M/2)^2/(0.79*R)^2$$

The pressure between them is as usual force per unit area, so we have,

$$P=F/A=(2^{(4/3)}/16pi).(M^2G/R^4)$$

19/02/1999

Free Fall Time scale, t_{ff}

This depends on distance r, mass and the gravitational constant.

$$F=M.(d^2r/dt^2)=-GM^2/r^2$$
$$d^2r/dt^2=-GM/r^2$$

Without precisly solving this, the average value is

$$d^2r/dt^2=-R/t^2 \text{ (approx)}$$

$$t_{ff}=(R^3/GM)^{1/2}$$

This is consistent with Kepler's third law.

We now apply this to our galaxy. In the galatic plane the density is þ=10^{-19} kg/m^3. This density is the density of material that would eventually collapse into itself and form a star. We also that,

$$R=(M/4pi\text{þ})^{1/3}$$

Given that the mass of the sun is $1.989*10^{30}$ kg the radius of a star like the sun would be $R=1.16*10^{16}$ m. Therefore the age of the sun (free fall time) would be 10^{14} years. This is how long it would take for intersteller dust to collapse into it's mutual center of mass to start the formation of the sun.

Gravitational Potential Energy time scale

Approaching star formation from this area leads us to the Kelvin-Helmholtz time scale, t_k.

The gravitational potential energy is found as,

$$\Omega=F.r=-Gm_1m_2/r$$

We can approximately say,

$$\Omega_{sun}=-GM^2/R=3.8*10^{41}\ J\ (approx)$$

The energy loss or louminosity of the sun is,

$$L_{sun}=3.8*10^{26}\ W$$

So the life time of the sun, given it's available energy and output rate is,

$$t_k=\Omega_{sun}/L_{sun}=10^{15}\ s=30*10^6\ years$$

This was in conflict with Darwinian evolution because a longer time scale was required for geographical time.

The Einstein time scale, t_E

Recall the relation,

$$E=mC^2$$

Eddington used this to determine the energy available in a star like the sun.

$$Esun=M_{sun}.C^2=1.8*10^{47}\ J$$

So the einstien time scale would be,

$$t_E=5*10^{20}\ s=1.5*10^{13}\ years.$$

This is in moe agreement with the requirement for Darwin's theories.

The Brightness of starts.

It all started 200 years BC!
Hipparchus invented the system of magnitudes to measure the brightness of stars. It roughly followed that a reference star was defined as magnitude 1 and then a star which is half as bright is magnitude 2. Mangitudes are calculated from putting the number 2.515 to the power of the star's magnitude.

Problems with the magnitude system is when we consider different wavelengths and use different detectors than the human eye (different systems *see* things differently).

There are two types of magnitude/luminosty systems used. **Absolute luminosity**(L) is the actual power emitted by the star. **Apparent luminosity**(l) is the power we observe at our position in space. The difference is that two different stars could have the same apparent luminosity by virtue of thier different distances from us.

Say we have two stars with magnitude difference m_2-m_1 and brightness ratio b_1/b_2. The two are related by,

$$m_2-m_1=2.5*\log(b_1/b_2)$$

So if we liken this to absolute (M) and apparent (m) magnitudes we can say,

$$M=m-5.\log(r/10)$$

Where r is in Parsecs.

Bolometric Magnitude

$$M_{bol}=M-BC$$

Where BC is the bolometric correction. This depends on the temperature of a star. For the sun, M=4.83 and M_{bol}=4.75. the correction is much larger for stars much hotter or colder than the sun. This bolometric magnitude is important because of the following relation,

$$M_{bol}=4.72-2.5*\log(L/L_{sun})$$

Sirius

Sirius has an apparent magnitude of -1.46, a radius of 2.7 pc and a temperature of aapproximately 10,000 K (BC=0.6). The absolute magnitude is M=m-5log(r/10)=1.4. So the Bolometric magnitude is 0.8. The ratio from Sirius's luminosity and the sun's luminosity is 40.

26/02/1999

Stars as black body sources

For even simple stellar models we need to know about certain propteries like; density, mass, diammeter, constituents and temperature. However we can account for many of these if we model stars as black body radiators. We can hence employ Plank's law,

$$U_k=(8.pi.c.h/lambda^5)[1/(exp(ch/lambda*kT)-1)]$$

energy per wavelength

So the total energy can be expressed as,

$$U=Integ\ (infin,0)\ \{U_k dlambda\} =(8pi^5k^4T^4)/(15c^3h^3)\ \ J/m^3$$

The black body radiated emittance R_{BB} is c/4 of above.

$$R_{BB}=2pi^5k^4T^4/15c^2h^3$$
$$=\grave{o}T^4\ \ W/m^2$$

Where ò is the Steffan-Boltzman constant and ò=5.669*10^{-8} W/m^2K^4.
We get U_{kmax} at some $lambda_{max}$ when,

$$d/dlambda\ \{lambda^5(exp(hc/lambda*kT)-1)\} =0$$

Which occurs when,

$$1-exp(hc/lambda*kT)=hc/(5*lambda*kT)$$

IF we write a general equation,

$$y=1-exp(A)=A/5$$

We can plot the intersection point,

fig 1

For the above, A=4.96, so that means

$$4.96=hc/lambda*kT$$

Therefore,

$$T.lambda_{max}=2.9*10^3\ Kelvin\text{-}Metres$$

For our sun, T=5800 K, lambda = 500 nm and R_{BB}=64 MW/m^2

see sheets ASTRO-2 & ASTRO-3 & ASTRO-4 & ASTRO-5

Solar observation

fig 2

In general intensity can be expressed as,

$$I=I_0exp(-\alpha dx)$$

From fig2,

$$x=(dR^2-2RdR)^{1/2}$$
$$dx=(dR^2-2RdR)^{1/2}-dR$$

So in this case we would write intensity as,

$$I=I_0\exp(-2RdR\alpha)$$

This is the formalism of the phenonemon known as ***limb darkening***.

Rather than using spectrometers with telescopes, colour filters can be used. This takes advantage that with black-body like emission there is a relationship between dominant colour and temperature. There are several filters primarilary used. The first is the U-filter, this *sees* wavelengths from 300nm to 400nm with the peak being 350 nm (the U is for ultraviolet). Next is the B (blue) filter which sees 380nm-550nm with the peak at 425 nm. Finally the V (visual) filter sees between 500nm-650nm with the peak at 550nm. In use, we have what is called a **two colour index** because to have a final value for a star it can be scanned by two filters and the difference of intensity from both filters taken. We use the B-V index and the U-B index. The B-V index for our sun is 0.62 which infers a temperature of 5800 Kelvin.

05/03/1999

Real stars cannot really be considered as black body radiators because they are not in equilibrium with their surroundings, they are releasing energy into space. Also there are some special spectra features which indicate emission and/or absorbtion processes happening in the star.

Doppler Broadening of spectral features due to star rotation.

If a star is rotating as we observe it. We see a particular (the dominant) wavelength from the central region of the star's body. However the extremities of the star are approaching on one side and receeding on the other side. We observe doppler shifts at each side of the star. Our sun has a rotation period of 25 days, it's radius is roughly $7*10^8$ m. So a point on it's perimeter would have a tangential velocity of ,

$$v= 2*pi*7*10^8/2.2*10^6=228867.9$$

We also know that,

$$dlambda/lambda=v/c$$

So we have,

$$dlambda/lambda=7*10^{6}$$

Hence at 1 µm we have dlambda=7 pm, this is readilly detectable.

Stellar Classification

There are seven classes, in order of temperature (hot to cool), O, B, A, F, G, K, M. These are sub-divided in to 10 groups, O0,…,O9, etc. The classes are sometimes recalled by the mnenomic, *oh be a fine girl kiss me*.

We wish to know about star's mass, temperature and size. All we have to use is the star's luminosity. We know that the black body radiation is proportional to the luminosity and we also have Steffan's law,

$$R_{BB}=\grave{o}T^4 \quad W/m^2$$

We also know that,

$L=R_{BB}$Therefore, the luminosity is proportional to the radius.

$$L=4piR^2\grave{o}T^4$$

$$R=(1/T^2)*(L/4pi\grave{o})^{1/2}$$

Betelguese for example has the follow quantities,

- $L=10^4$ the luminosity of the sun
- T=3000 Kelvin
- $R=2.6*10^{11}$ m

$$L_{sun}=4pi*R_{sun}^2\grave{o}T_{sun}^4$$

$$L/L_{sun}=(R/R_{sun})^2*(T/T_{sun})^4$$

$$R/R_{sun}=(T_{sun}/T)^2*(L/L_{sun})^{1/2}$$

The relationship between mass and luminosity

Masses can be determined from the characteristics of binary systems. We can use Kepler's third law,

$$a^3/p^2=(G/4pi^2)(M_1+M_2)$$

(Where a is the distance between the bodies and p is the orbital period. For a binary system we define a as,

$$a=a_1+a_2$$

Where a_1 and a_2 are the respective distances from the mutual centre of mass.

From theory the is a relationship between luminosity and mass, it is of the form,

$$L=CM^x$$

So there is some logarithmic proportionality between the two quantities. C and x are constants, which are different for every star. If for example we have a mass between 1 and 10 solar masses, and x is between 3 and 4.5, we can say (changing constants),

$$\mathrm{Log}(L/L_{sun})=\beta+\alpha\log(M/M_{sun})$$

	α	β	
$M<0.5M_{sun}$	2.85	-.15	
$0.5M_{sun}<M<2.5M_{sun}$		3.6	0.073
M>2.5Msun	2.91	0.499	

$$M\sim M_{sun},\quad L/L_{sun}=(M/M_{sun})^{3.5}$$

12/03/1999

Typically in the center of a star we need temperatures or the order 10^7K, and pressure of the order 10^{16} N/m^2 and a density of the order 10^5 Kg/m^3. As the atoms interact with each other in such conditions, we can plot their potential energy with separation,

fig 1

From kinetic theory we can use,

$$P=nkT$$
$$(n=10^{32}\ m^{-3})$$
and,
$$cbar=(3kT/m)^{1/2}\sim 5e5\ m/s$$

cbar=rms speed. We can also calculate the classical energy of the particles,

$$E_{kin}=\tfrac{1}{2}mcbar^2=(3/2)kT=2e\text{-}16\ J$$

Since for a star to form we require E_c=8e-13 J (see fig 1). Fortunetly this is only a classical consideration, if we account for nuclear effects. We now consider another part of the atomic energy.

Nuclear Binding Energy

Let M_n be the neutron mass and M_p be the proton mass. We would assume the mass of the nucleus would simply be,

$$M=M_n+M_p$$

This is not observed, there is always some difference, which is called a *mass defect*. Using Einstein's relation, we can define the *binding energy*.

$$B=(M_o-M_x)C^2$$

If we consider taking a helium nucleus and converting it into 4 Hydrogens. The mass of Helium nuclei is 4.0039 amu and the mass of a proton is 1.0081 amu. with 1 amu = 1.66e-27. Assuming little difference between protons and neutrons, the difference in mass of four hydrogens and one helium is 0.0285 amu so the energy is,

$$dmC^2=4.3579e\text{-}12 \text{ J } (\sim 25 \text{ MeV})$$

So going from hydrogens to heliums, 6e14 J/Kg of energy would have to be released. So if all of the sun was made of hydrogen and it completely converted this hydrogen into helium it would release 10^{45} Joules. Which gives teh luminosity of the sun to be 4e26 Watts, so the *fuel supply* of the sun would last for 10^{11} years. So for stars where the mass is less than twice that of our sun the main energy supply is "hydrogen burning" (H -> He). For such processes, called the p-p chain, we have the following nuclear reactions,

$$^1H+^1H \rightarrow ^2D + e^+ + \upsilon$$

e^+ is a positron and υ is a neutrino.

$$^2D+^1H \rightarrow ^3He + \gamma$$
$$^3He + ^3He \rightarrow ^4He + ^1H + ^1H$$

This final step gives back some hydrogen with which the cycle can start all over again. There are other processes possible, some are outlined on the "Fusion reaction in stellar interiors" sheet.

16/4/1999

Star Brith and Structure

Two useful questions can be stated here;
1) Why can't we make micro-stars in the lab? &
2) Why are there no mega-stars ?

In answer, there are limits to the size of a star, determined by it's

mass, this comes from;
1) The ability of gas cloud to condense,
2) Stability,
3) Temperature for thermo-nuclear fusion (10^7 K).

The first two arise from having a gravitationally bound system (temperature is a consequence of condensation). The gas cloud becomes gravitationally bound when the gravitational potential energy becomes greater than the kinetic energy of the particles.

Let us now consider the condenstation of a cloud of radius R, containing N particles each of mass m, at a temperature T. We can also say, M=Nm, where M is obviously the total mass of the system. We define the gravitational potential energy,

$$E_{GR}=-f(GM^2/R)$$

f is equal to 3/5 for a uniform spherical cloud. It is great than 3/5 with a dense center, therefore f approximately 1. Hence,

$$E_{GR}=-GM^2/R \quad (1)$$

The internal kinetic energy is,

$$E_{KE}=(2/3)NkT \quad (2)$$

For condensation we require,

$$|E_{GR}|>E_{KE}$$

So,

$$GM^2/R>3NkT/2$$

An astrophysicist called Jean defined a set of minimum condition required for the initial contraction of a star. From his work we can state,

$$MNm>3NkTR/2G$$
$$M_J>3kTR/2Gm \quad (3)$$

and,

$$þ_J>(3/2)(kT/Gm)(1/4piR^2)$$
$$=(3/4piM^2)(3kT/2Gm)^3$$

$þ_J$ is more easily achieved if M is large. For a Hydrogen cloud at 20 K, and $M=10^3$ solar masses = $2*10^{33}$ Kg which gives us a critical density of 10^{-22} Kg/m^3 which is 10^5 molecules per cubic meter. However if M is one solar mass, $þ_J$ becomes 10^{11} molecules per cubic meter.

At this early stage we have a **protostar**. This is still not a table system.

We need to achieve **Hydrostatic Equilibrium**. The star is approximately spherical, from using it's radius we can determine it's ; pressure, density, temperature and m. We would expect these quantities to be greater towards the centre of the star. We would expect the following,

$$dP/dr=-Gmþ/r^2 \qquad (1)$$

The negative sign indicates that the pressure drops off as we go out from the centre of the star. We can also write,

$$dm/dr=4pir^2þ \qquad (2)$$

If we divide equation one by equation two, we get,

$$dP/dm=-GM/4pir^4 \qquad (3)$$

If we take equation one and divide by the volume and integrate from r=0 to r=R we obtain,

$$\text{Integ } (0,R)\ [4pir^3(dP/dr)dr]=\text{Integ } (0,R)\ [(Gmþ4pir^2/r)dr] \qquad (4)$$

The right and side is actually the gravitational potential energy. The left hand side is the volume average pressure, P^*, multilplie by -3V. So we can write,

$$-3P^*V=E_{GR}$$

Hence we can write,

$$P^*=E_{GR}/3V \qquad (5)$$

This means that the average pressure required to support a gravitationally bound system is 1/3 of the stored gravitational potential energy density. This is known as the Virial Theorem.

If we consider classical (non-relativistic) gas, we can state,

$$P^*=(1/3)mv^2n=(2/3)n(½mv^2)$$

we can even say,

$$P^*=(2/3)(E_{KE}/V) \qquad (6)$$

If we now substitute equation 6 into equaiton 5,

$$(2/3)(E_{KE}/V)=-(1/3)(E_{GR}/V)$$

Therefore we can write,

$$2E_{KE}+E_{GR}=0 \qquad (7)$$

23/04/1999

if no excited internal degrees of freedom, than all internal energy due to translation,

$$\mathbf{E_{total}=E_{ke}+E_{gr}}$$
$$\mathbf{E_{total}=-E_{ke} \text{ and } E_{total}=E_{gr}/2}$$

Slowly evolving star close to hydrostatic equilibrium. eg, a 1% decrease in E_{total} gives a 2% decrease in E_{gr} and 1% increase in E_{ke}. The average pressure,

$$\mathbf{P^{*}=NkT/V=2E_{ke}/3V}$$
$$\mathbf{T=2E_{ke}/3Nk}$$

Therefore,

decrease in total energy makes a hotter star

½E_{gr} gives rise to heating and ½E_{gr} tends to radiated energy. However if thermonuclear energy maintains L(uminosity).

$$\mathbf{E_{ke}+E_{gr}=constant}$$

From the Jean's density,

$$\mathbf{þ_J=(3/4piM^2)(3kT/2Gm)^3}$$

This relation implies a cloud of temperature 20K and a density of þ~10^{-16} Kg/m^3. This allows independant contraction of fragments M~M_{sun}. At this stage our proto-star has a radius of about 10^{15} m (this is about a million times larger than the sun). For furter collapse we need an energy sink (as to not increase the random motion of particles and hence increase the pressure). Dissociation and ionisation of the gas molecules would serve this purpose. We will assume in our analysis that the gas is all Hydrogen. The dissociation energy of hydrogen is 4.5 eV per molecule (7.209*10^{-19} J per molecule). The ionization energy of Hydrogen is 13.6 eV per atom (2.179*10^{-18} J pe atom). The energy required to dissociate and ionize the whole proto-star is then,

$$\mathbf{(M2m_h)E_D + (M/m_h)E_i}$$

In going from initial radius R1 to final radius R2,

$$\mathbf{(GM^2/R1)(GM^2/R2) \sim= (M2m_h)E_D + (M/m_h)E_i}$$

So for one solar mass, the total energy required is approximatley 3*10^{39} J. The time of collapse is calculated using the free fall method (as described earlier), to collapse from 10^6 solar radii to 10^2 solar radii.

This would hence take 20,000 years !

We use virial theorem and kinetic theory to estimate the temperature. The gravitational potential energy is said to be (since there is a great difference between the two radii, we neglect the smaller one),

$$E_{gr}=-GM^2/R=-(M/m_h)((E_D/2)+E_I)$$
$$E_{ke}=(3/2)kT[(M/m_h^+)+(M/m_h^-)]$$
$$=3kT(M/m_h)$$

$$2E_{ke}+E_{gr}=0$$

$$6kt(M/m_h)=(M/m_h)[½E_D+E_I]$$

$$kT=(E_D/12)+(E_I/6)\sim=2.6\ eV$$

$$T \sim 30,000\ K$$

Note that this temperature is independant of the mass, but this is too cold to become a star. The time scale for this is about 10^7 years. Now we must consider the process of heating the star again.

Consider a <u>Degenerate gas</u>.
Gravitational collapse may be halted by a degenerate gas (cold, dense) hence we wish to finda way around this. The condition for degeneracy is,

distance between electrons ~ DeBroglie wavelength

$$lambda_B=h/p$$

The kinetic energy of a classical gas is ~ kT

$$p \sim (m_e kT)^{½}$$
$$lambda_B \sim= h/(m_e kT)^{½}$$

To avoid dengeneracy:

$$p_{quantum}<<m^*/lambda_B^3 \sim= m^*(m_e kT)^{3/2}/h^3$$

m^* is approximately 0.5 amu. From classical theory,

$$kT \sim GMm^*/3R \sim= Gm^*M^{2/3}p^{1/3}$$
$$T\ a\ p^{1/3}$$

The temperature at which electrons become degenerate is,

$$kT = [G^2m^{*8/3}m_e/h^2]M^{4/3}$$
$$T\ a\ M^{4/3}$$

For one solar mass, $2*10^3$ Kg, kT ~ 1 KeV which gives us 10^7 K.

A STAR IS BORN !!!

The minimum is 0.08 solar masses, this is why we can't make a star in the lab! If the mass if less than this minimum value gives us **Brown dwarfs**.

07/05/1999

Now let us consider stars with a mass much greater than that of our own sun ($M>>M_{sun}$). Gravity is opposed by photon pressure not gas pressure. Recall the Virial theorem,

$$P^*=-(1/3)E_{GR}/V \qquad (1)$$

and for classical gas,

$$P^*=(2/3)(E_{KE}/V)$$

Hence for photons,

$$P^*=(1/3)n(pc)$$

p is momentum and c is the speed of light. We then get,

$$P^*=(1/3)(E_{KE}/V) \qquad (2)$$

Hence for Hydrostatic equilibirum we require,

$$(1/3)(E_{KE}/V))=-(1/3)(E_{GR}/V)$$
$$E_{KE}+E_{GR}=0$$

Hydrostatic equilibrium is only possible if the binding energy tends towards zero, hence very large stars are unstable.

Stellar Structure

There are various models. There are may *interesting* quantities to look at such as,

$$dm/dr\ ,\ dT/dr\ ,\ d\text{þ}/dr\ ,\ dP/dr\ ,\ dL/dr$$

We would an equation of state such as P(r,þ,T). We need to make certain assumptions when forming a model. We assume spherical symmetry, Hydrostatic equilibium, local thermal equilibrium, energy transport mechanisms.

We will now consider a model (by P H Key, our lecturer) which doesn't acually work, but should be illuminating all the same.

Key Rough Astro Physics (KRAP) Model for steller Temperature

Consider a spherical star of radius R and mass approximately the same as our sun and it is in Hydrostatic equilibirum. We also state that the core temperature is much greater than the surface temperature. The luminosity supplied from the core and is fixed.

$$T_c >> T_s$$

We also assume that the core is much much smaller than the total radius.

We next consider the energy flux, Q, throug the star in units of energy per unit area per unit time. The energy flux starts at the core at a value Q_c and arrives at the surface with a value of Q_s. We can calculate these hence,

$$Q_c = L/4pir_c^2$$
$$Q_s = L//4ipR^2$$
$$Q_s R^2 = Q_c r_c^2$$

We can also generally say,

$$Q = CT$$

where C is the thermal capacity, so we now say,

$$C_s T_s R^2 = C_c T_c r_c^2$$

If $C_s=C_c$ we have an isotropic material. Now we can say,

$$r_c/R = (T_s/T_c)^{1/2}$$

Now for the sun we state,

$$r_c = 0.025*R$$

This is an estimate which is not very realistic. This s calculated from the assumptions that the temperature linearly decreases with radius, there is a constant density and pressure with radius and that steller mass is directly proportional to radius. These are unrealistic, but there is a justifacation, and they lead us to a better model.

If þ s constant,

$$M/V = m_c/V_v$$

and,

$$m_c=Mr_c^3/R^3$$

For our sun we say,

$$m_{c\text{-}sun} \sim= 3e25 \text{ Kg}$$

Hydrogen to Helium reactions produces 614 J/Kg, so m_c at r_c=0.025*R_{sun} gives 1.8e4 J. But L_{sun}=3.6e26 W. Therefore the life time of the sum should be 1e6 years (but this does not agree with geological time!). If r_c=0.25*R_{sun} we have 1e9 years for lifetime. This is achieved by assuming the radius is an order of magnitude larger, we get a change in magnitude of three in the lifetime, this gives us a more realistic value!

(See sheet for a theoretical model of the sun)

Energy (heat) Transport in Stars

The one dimensional heat conduction equation tells us that a small quantity of heat dq is,

$$dq/dt=-K(dT/dr)$$

(K is thermal 'conductivity', which should be regarded as the thermal transport coefficient)

Case 1 : Random motion of Particles.
We need to consider the particle to particle collisions. If we have n particles per unit volume, each travelling at an RMS speed v^* (m/s), each particle has a mass m (assumnig they are all the same), they all have a heat capacity C, and there mean free path (mfp) of l^* between collisions. We now say,

$$K=(1/3)nv^*mCl^* \text{ W/m/K}$$

We know what the RMS speed is from kinetc theory,

$$v^*=3kT/m$$

The mfp is a problem to work out. For various types of collisions we need to pick one that works well for us in terms of heat conduction, electron electron and electron ion collisions are not good, but phton transport of energy is more efficient.

14/05/1999

Radiative Diffusion

This is a major energy transport method. It is the transport of energy by photons. Consider the sun radiating energy with luminosity L. Using a black body model we can say,

$$L \alpha T^4$$

Wein's Law

For a balck body model at a specific temperature, we know that if we plot Luminosity against wavelength, there is a single peak and we can calculate it as,

$$Lambda_{max}=2.9e\text{-}3/T \text{ metres.}$$

The solar surface is approximately T_s ~= 6000 K, the photon energy is then kT ~= 0.5 eV, which gives visible radiation, as would be expected! The solar core however, is roughly 6e6 Kelvin, so, kT ~= 0.5 KeV, which is X-Ray energy. So how do we not get many X-Ray photons from the sun ? The photons inside the sun are abosrbed and re-emitted and scattered which causes the phtonic energy to drop. The scattering process causes photons to take a longer path out of the sun. So we can define a mean free path for photons,

$$l^{*2}=Sum(l^2)/N$$

Where l is each uninterupted path and N is the number of collisions. For instance if a photon travels from the inside to the outside of a star, along two paths (l_1 and l_2) for a final displacement R, the radius we have $R^2=l_1{}^2+l_2{}^2$. If we say that the photons take a *random walk*, we can work out the time taken for a photon to leave the sun,

$$t_{RW}=Nl^*/C=R^2/l^*C$$

Where is is the speed of light. We can also say,

$$t_{direct}=R/C$$

Therefore,

$$t_{RW}=t_{direct} * R/l^*$$

Hence we can make a similar deduction for the surface and core temperatures,

$$T_s=(l^*/R)^{1/4} * T_c$$

So with a guess value of l^* being 1mm, then we have t_{RW}= 50,000 years. However l^* wil vary with density and probability of interaction with an ion or electron. Note,

$$l^*=(1/(n_iS_i+n_eS_e))$$

Wheren is number density for ions and electrons respectively and S is the cross section, likewise. n is proportional to the mass density. We have another relation too,

$$l^*=1/þk$$

Where k is the opacity. We now return to calculating the radiative difusion of energy.

"Conductivity heat transer coefficient" = K

$$K=(16/3)ò(T^3/þk)$$

$$dq/dt = -(16/3)ò(T^3/þk)(dT/dr)$$

Now we consider bulk transfers of energy,

Convection

Here we primarily have large loops of heat flow. Convection can occur when the temperature gradient dT/dr is,

$$dT/dr = ((y-1)/y)(P/T)(dP/dr)$$

For three degrees of translational freedom, y=5/3, hence,

$$dT/dr=(2/5)(P/T)(dP/dr)$$

Convection is good at heat transfer and will occur when the the temperature gradient is sufficiently high. In the sun, in the inner 80% of the radius, heat is transfered by radiative diffusion and in the outer 20% of the radius heat transfer is by convection.

Equation of Stellar Structure

Hydrostatic Equilibrium,

$$dP/dr=-Gm(r)þ(r)/r^2$$

(1)

dm/dr=4pir²þ(r)
2)

Thermodnamic Equilibrium

dT/dr=-(3/16ò)k(r)þ(r)L(r)/(4pi²(T(r))³)
dL/dr=4pir²*E*(r)

E(r) is the energy density.

Clayton in 1968 said that we shoulf just guess at the form of dP/dr.
We can assume that dP/dr -> 0 at r=0 and r=R.

see sheet

That's all folks !!! :-)